Holography and Cosmological Singularities

Ben Craps

Vrije Universiteit Brussel & International Solvay Institutes

Based on work with A. Bernamonti, T. Hertog, and N. Turok

International Solvay Institutes Brussels

Fundaments Of Gravity
LMU, Munich
April 16, 2010

Vrije Universiteit Brussel

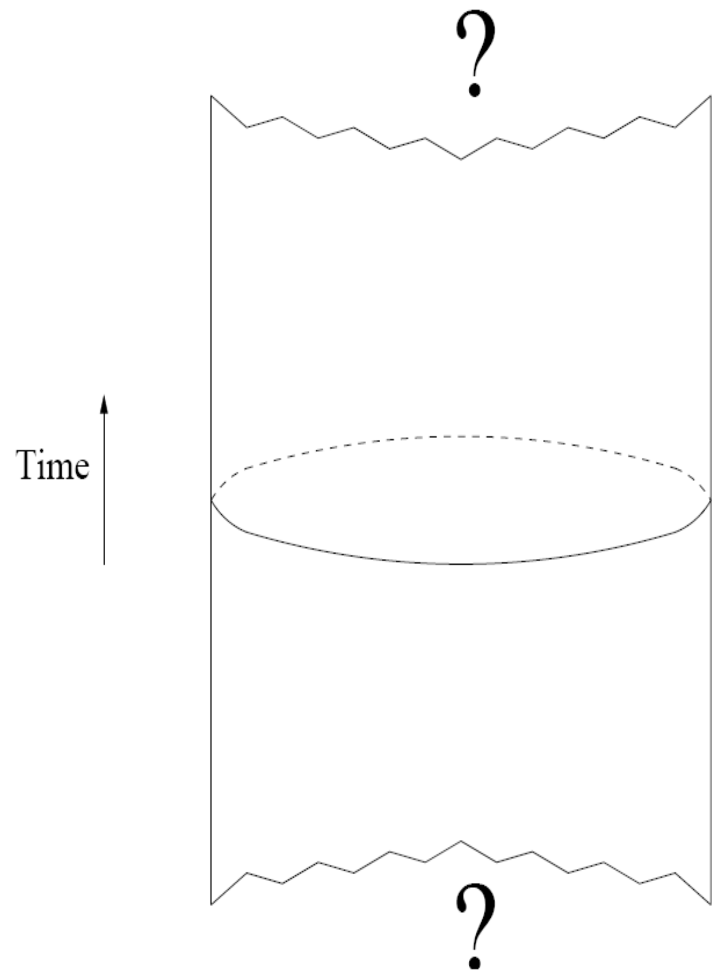
Motivations for study of cosmological singularities

- Conceptual questions: nature of big bang, emergence of space and time?
- Big crunch/big bang cosmologies: propagation of perturbations through singularity?
- Inflation: what sets initial conditions?

AdS cosmologies: basic idea

Starting point: supergravity solutions in which smooth, asymptotically AdS initial data evolve to a big crunch singularity in the future (and to a big bang singularity in the past).

Can a dual gauge theory be used to study the singularity in quantum gravity?



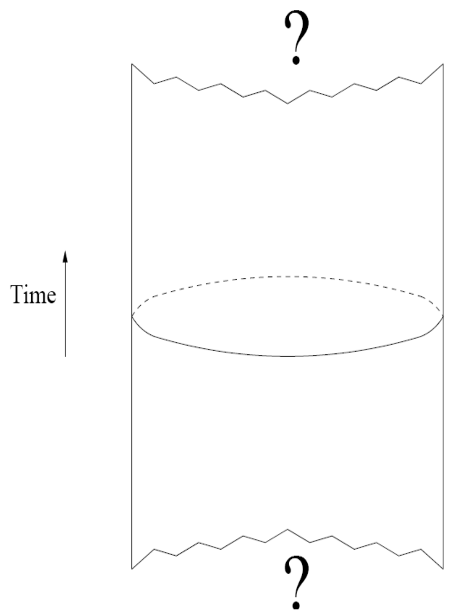
Hertog, Horowitz

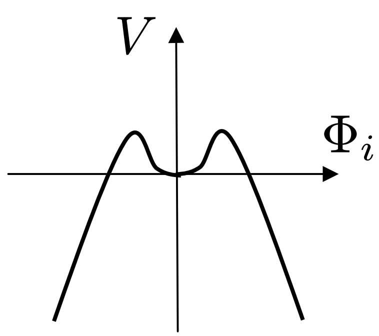
AdS cosmologies: basic idea

Modified (non-susy) boundary conditions: smooth initial data that evolve into big crunch

⟺

Potential unbounded from below in dual field theory; operator reaches infinity in finite time

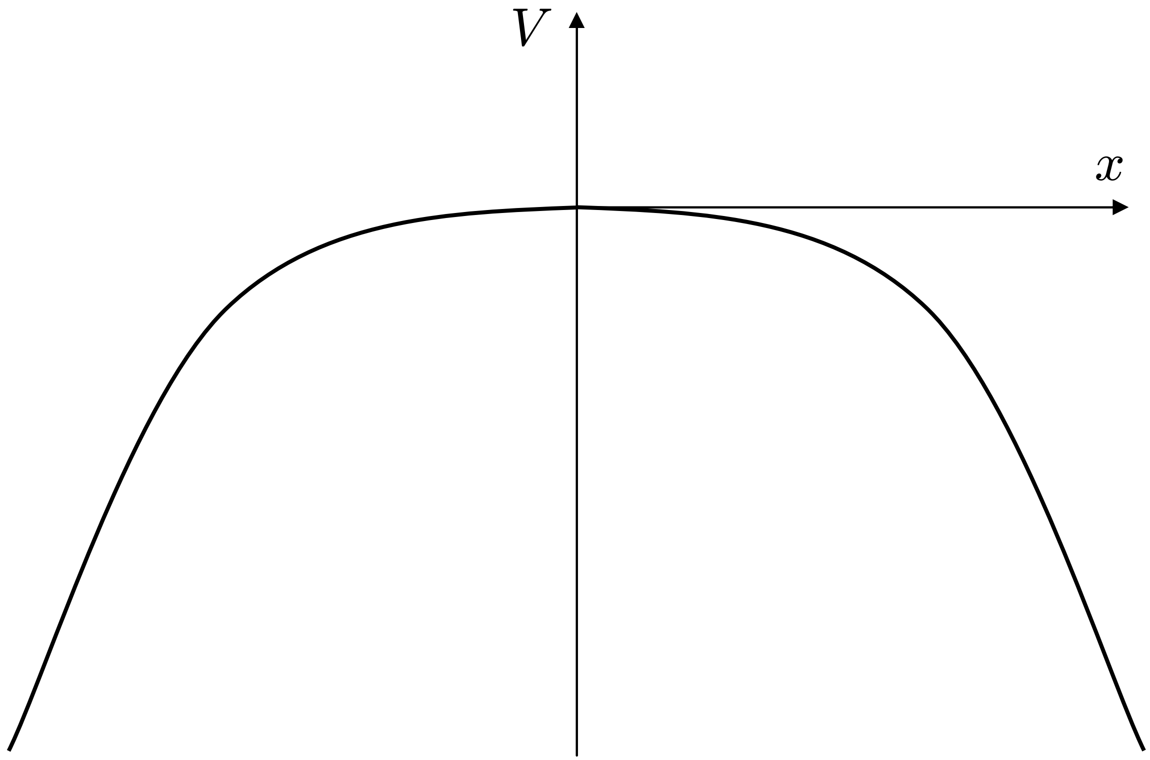




Goal: learn something about cosmological singularities (in the bulk theory) by studying unbounded potentials (in the boundary theory)

Hertog, Horowitz

Beyond the singularity? Self-adjoint extensions



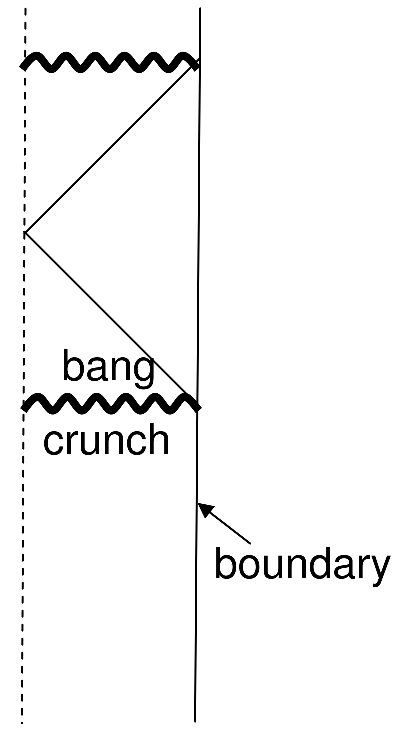
- QM: self-adjoint extension → unitary time evolution
- Possible in QFT??

BC, Hertog, Turok

Big crunch/big bang cosmology?

Program (in principle):

- Take a state in the bulk theory
  (with modified boundary conditions)
- Translate to state in dual boundary theory
  (with unbounded potential)
- Evolve state through singularity using
  self-adjoint extension
- Translate evolved state back to state in bulk theory



Can this be implemented in a controlled way?

BC, Hertog, Turok

Outline

I. AdS_5 cosmology and N=4 SYM

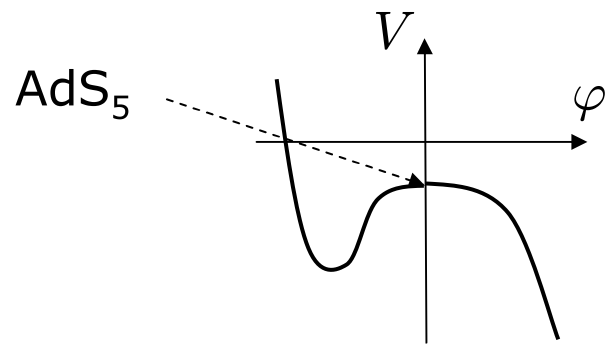
II. AdS_4 cosmology and ABJM theory

The bulk theory: AdS$_5$ cosmology

IIB sugra on S^5

↓ consistent truncation

5d $g_{\mu\nu}$ + scalar φ



$$m^2 = m^2_{BF}$$

AAdS: $$ds^2 \sim -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_3^2 \qquad (r \to \infty)$$

$$\varphi(r) \sim \alpha(t,\Omega)\,\frac{\ln r}{r^2} + \frac{\beta(t,\Omega)}{r^2} \qquad (r \to \infty)$$

Boundary condition: $\alpha = f\beta \quad (f > 0)$

crunch from smooth initial data

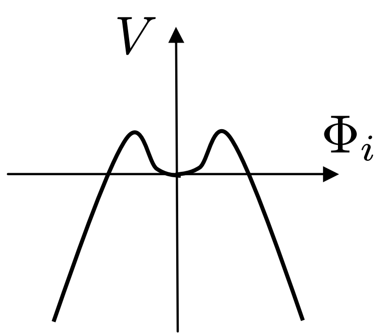
Hertog, Horowitz; BC, Hertog, Turok

The dual field theory: N=4 SYM

$\mathcal{N}=4$ supersymmetric SU(N) Yang-Mills theory on $\mathbb{R}\times S^3$

- 6 adjoint scalars Φ_i, in fundamental of $SO(6)_R$
- operator dual to bulk scalar φ: $\mathcal{O}\sim\frac{1}{N}\mathrm{Tr}\left[\Phi_1^2-\frac{1}{5}\sum_{i=2}^{6}\Phi_i^2\right]$
- boundary conditions $\alpha=f\beta \quad (f>0)$

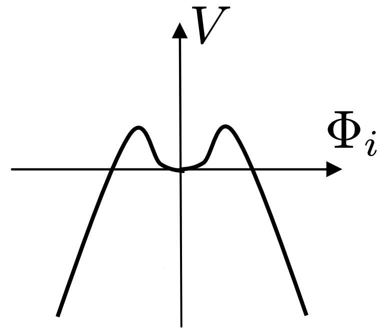
→ $$S=S_{SYM}-\text{ conf. coupl. }+\frac{f}{2}\int\mathcal{O}^2$$



Aharony, Berkooz, Silverstein; Witten; Berkooz, Sever, Shomer

The dual field theory: quantum effective potential

$$S = S_{SYM} - \text{ conf. coupl. } + \frac{f}{2}\int \mathcal{O}^2$$



- One-loop (Coleman-Weinberg) effective potential:

$$V = -\frac{f_{\mathcal{O}}}{2}\mathcal{O}^2 \quad \text{with} \quad f_{\mathcal{O}} = \frac{2}{\log(\mathcal{O}/M^2)}$$

→ Asymptotically free, no turning around

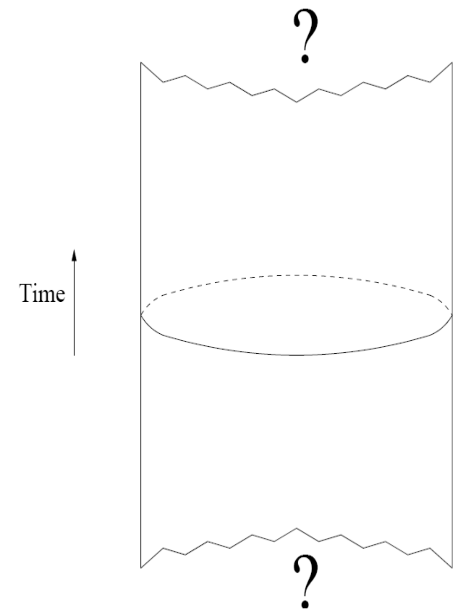
- Need small $g^2_{YM}N$ to have complete control over field theory
- Beta function for f is one-loop exact in large N limit → running of f extends to large 't Hooft coupling

Consistent truncation

Solution for 5d metric $g_{\mu\nu}$ and scalar φ → solution to 10d IIB sugra

$$\begin{cases} ds^2_{10} = \ldots \\ \hat{G}_5 = \ldots \end{cases}$$

Cvetic, Lu, Pope, Sadrzadeh, Tran



φ : quadrupole deformation of S^5

Big crunch solution: $\varphi \to \infty$ and $g_{\mu\nu} \to$ curvature singularity in finite global time

Coulomb branch solutions

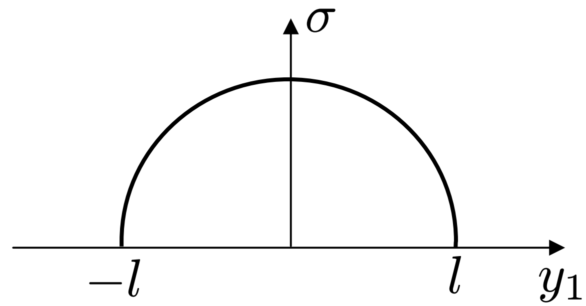
Near-horizon limit of distribution σ of parallel D3-branes:

$$ds^2_{10} = \frac{1}{\sqrt{H}}(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \sqrt{H}\sum_{i=1}^{6} dy_i^2$$

$$H = \int d^6w \; \sigma(\vec{w}) \; \frac{1}{|\vec{y} - \vec{w}|^4}$$

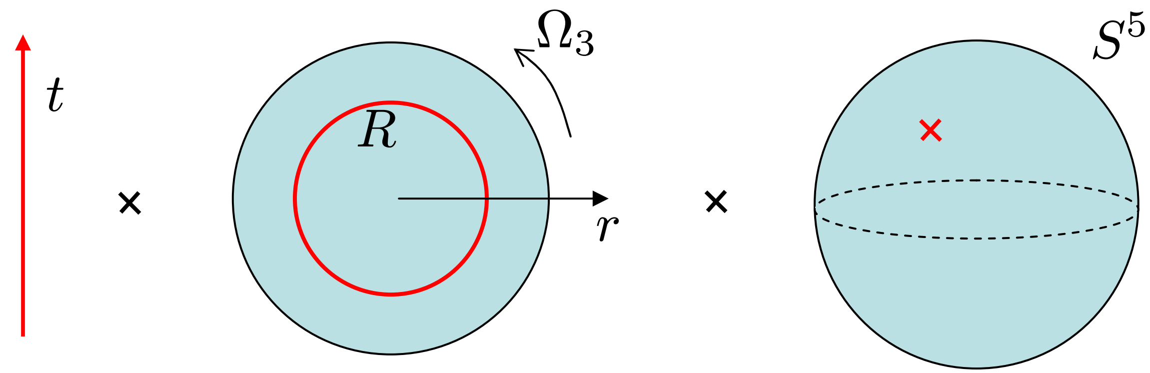
Fits in consistent truncation ←→ $\sigma(\vec{w}) = \frac{2}{\pi l^2}\sqrt{l^2 - w_1^2}\;\delta(w_2)\ldots\delta(w_6)$

D3-branes spread out over 1d interval of size l



Freedman, Gubser, Pilch, Warner

Effective potential for large spherical D3 in AAdS$_5$



Consider large spherical probe D3-branes in $AdS_5 \times S^5$ (can nucleate or be present in initial state)

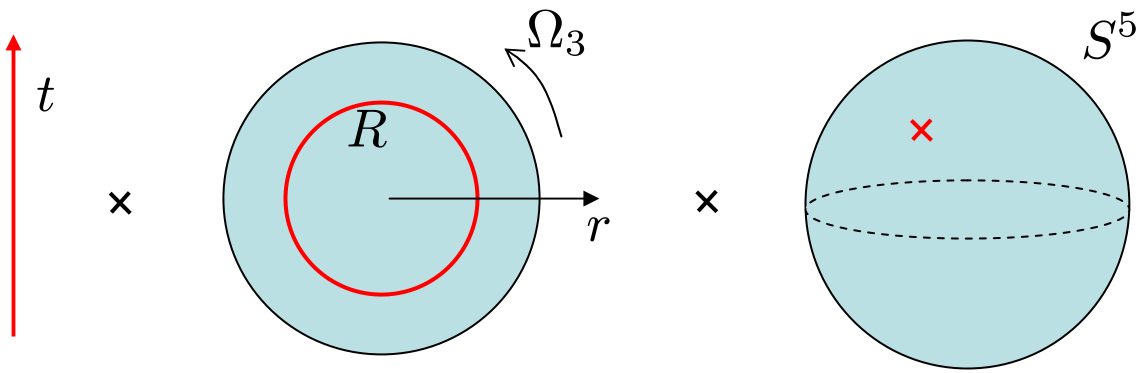
"Disk" contribution to effective potential (independent of b.c.):

$$S = S_{DBI} + S_{WZ}$$

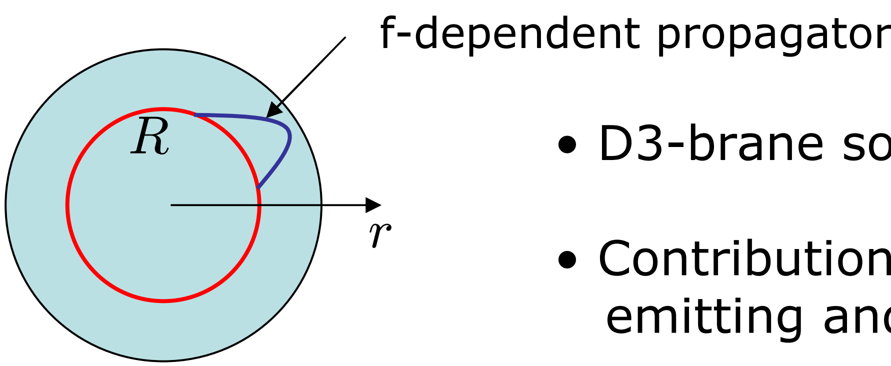
$$V \sim \tau_3 R^3 \left(\sqrt{1 + \frac{R^2}{R_{AdS}^2}} - \frac{R}{R_{AdS}} \right) \sim \tau_3 R_{AdS} R^2 \qquad (R \gg R_{AdS})$$

→ Conformal coupling in $\mathcal{N} = 4$ SYM on $\mathbb{R} \times S^3$

Effective potential for large spherical D3 in $AAdS_5$



"Cylinder" contribution to effective potential:

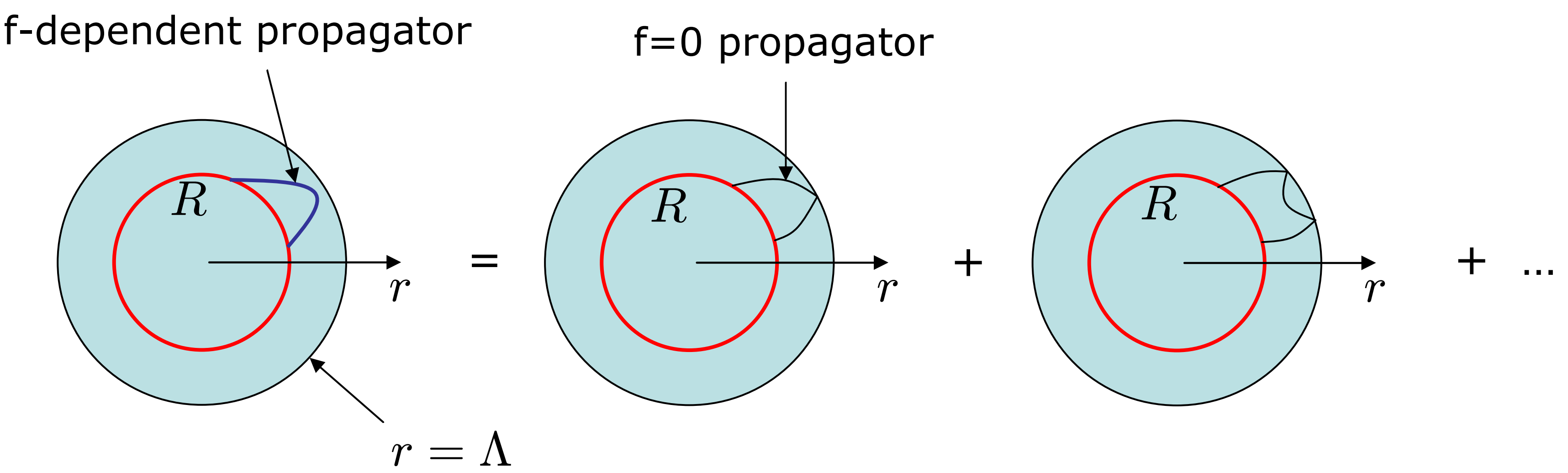


- D3-brane sources φ
- Contribution to effective potential from D3 emitting and reabsorbing φ
- Sensitive to boundary condition on φ

Bernamonti, BC

Effective potential for large spherical D3 in $AAdS_5$

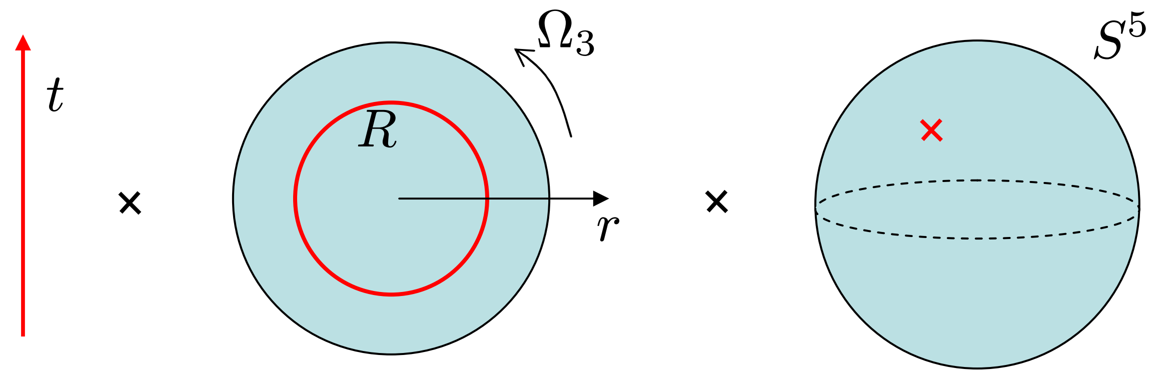
$$\delta S_{bndy} \sim \int_{r=\Lambda} d^4x \sqrt{g_{bndy}} \frac{f}{2\left[1 + f \ln\left(\frac{\Lambda}{R_{AdS}}\right)\right]} \varphi^2$$



Can work with f=0 propagator, but then needs to take boundary interaction explicitly into account.

Bernamonti, BC

Effective potential for large spherical D3 in $AAdS_5$



Result:

$$\delta V_{eff} \sim f R^4 \left(\cos^2 \xi - \frac{1}{5} \sin^2 \xi \right)^2$$

$$\sim -\frac{f}{N^2} \left(\phi_1^2 - \frac{1}{5} \sum_{i=2}^{6} \phi_i^2 \right)^2$$

$$d\Omega_5^2 = d\xi^2 + \sin^2 \xi d\Omega_4^2$$

$$\phi_1 \sim R \cos \xi$$
$$\phi_2 \sim R \sin \xi \cos \Omega_1$$
$$\vdots$$

Bernamonti, BC

Implications of D3-brane interpretation

- Large spherical D3-branes feel a potential pulling them to infinite radius
    - ➢ can nucleate in AdS
    - ➢ can be present in initial state
- D3's are domain walls: 5-form flux at fixed r decreases

  → decrease of effective $g_{YM}^2 N$

  → field theory weakly coupled near singularity?
- Relation with sugra solutions: $\varphi \sim \frac{l^2}{r^2}$
- Subtlety: $\int_{S^5} \hat{G}_5 \sim N$ for sugra solutions. Cf. geometric transition

Bernamonti, BC

Implications of D3-brane interpretation

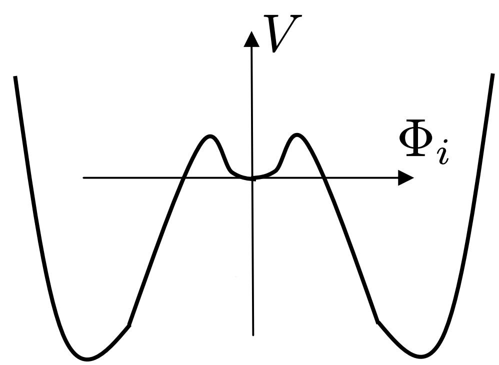
- Status of AdS/CFT with modified boundary conditions?

  - Lack of decoupling?
  - Can it be obtained as near-horizon limit of D3-branes in non-trivial background?

- Regularizing the field theory potential

  - Black hole, singularity shielded by horizon!

Hertog, Horowitz; Barbon, Rabinovici



Problems of AdS_5 cosmologies

- Self-adjoint extensions of "brick wall" type do not straightforwardly extend to field theory (breakdown of ultralocality)

  → Possible cure: "analytic self-adjoint extensions"

- Running coupling leads to large particle creation and loss of computational control

  → Appears to be better in AdS_4 model

BC, Hertog, Turok, in progress

Outline

I. AdS_5 cosmology and N=4 SYM

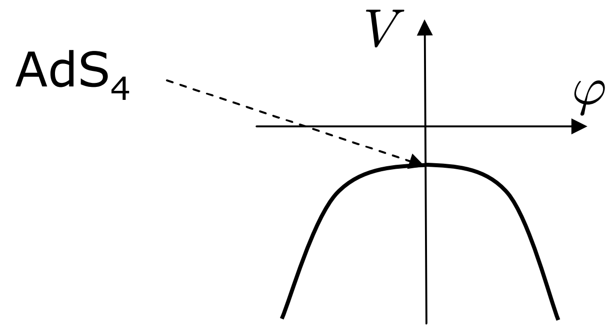
II. AdS_4 cosmology and ABJM theory

The bulk theory: AdS$_4$ cosmology

11d sugra on $S^7/\mathbb{Z}_k$

↓ consistent truncation

4d $g_{\mu\nu}$ + scalar φ



$$m_{BF}^2 < m^2 < m_{BF}^2 + 1$$

<u>AAdS</u>: $$ds^2 \sim -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_2^2 \qquad (r \to \infty)$$

$$\varphi(r) \sim \frac{\alpha(t,\Omega)}{r} + \frac{\beta(t,\Omega)}{r^2} \qquad (r \to \infty)$$

<u>Boundary condition</u>: $\beta = -h\alpha^2$

AdS invariant; crunch from smooth (instanton) initial data

Hertog, Horowitz

The dual field theory: ABJM theory

$\mathcal{N}=6$ superconformal U(N) x U(N) Chern-Simons theory, levels k resp. -k

- gauge fields $A_\mu,\ \hat{A}_\mu$
- scalars $Y^A,\ A=1,\ldots,4$, in $\begin{cases} \text{fundamental of } SU(4)_R \\ (N,\bar{N}) \text{ of } U(N)\times U(N) \end{cases}$

  sextic single trace potential

- describes N M2-branes on $\mathbb{C}^4/\mathbb{Z}_k$: $y^A \to e^{\frac{2\pi i}{k}} y^A$
- 't Hooft limit: $N\to\infty,\ N/k$ fixed (weakly coupled IIA string theory)
- operator dual to bulk scalar φ : $\mathcal{O} \sim \frac{1}{N^2}\mathrm{Tr}(Y_1 Y_1^\dagger - Y_2 Y_2^\dagger)$ BC, Hertog, Turok

Aharony, Bergman, Jafferis, Maldacena

The dual field theory: modified boundary conditions

$\beta = -h\alpha^2$

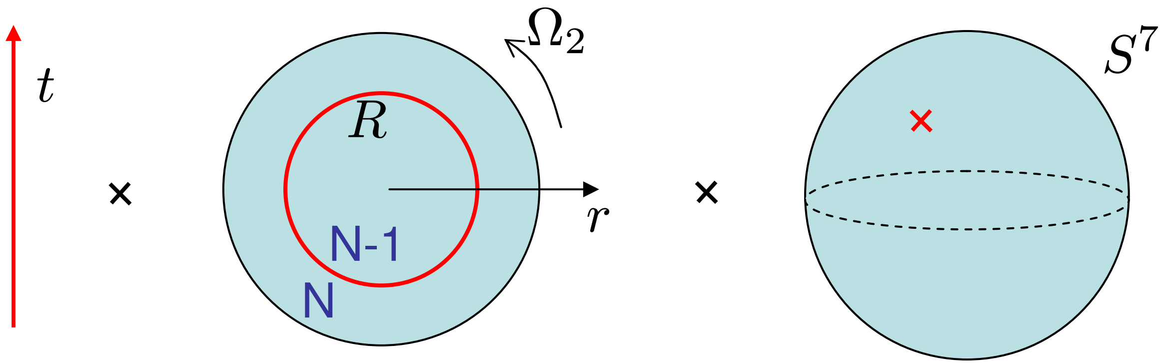
$$\varphi(r) \;\sim\; \frac{\alpha(t,\Omega)}{r} + \frac{\beta(t,\Omega)}{r^2} \qquad (r \to \infty)$$

$$S = S_{ABJM} + \text{ conf. coupl. } + \frac{h}{N^4}\left[\mathrm{Tr}(Y_1 Y_1^\dagger - Y_2 Y_2^\dagger)\right]^3$$

- conformal in planar limit
- reduces to $O(2N^2) \times O(2N^2)$ vector model in weak coupling limit $N \to \infty,\ N/k \to 0$

BC, Hertog, Turok

M2-brane interpretation of the instability



Bernamonti, BC

Consider spherical M2-branes in $AdS_4 \times S^7$ (can nucleate or be present in initial state)

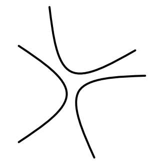
Susy b.c. → quadratic potential for canonically normalized field $\phi \equiv \sqrt{R}$ of spherical M2 (conformal coupling) → spherical branes shrink and annihilate

Modified b.c. → negative sextic term added to potential for $\phi \equiv \sqrt{R}$
→ large spherical M2-branes pulled to infinite radius in finite time

M2's are domain walls → 4-form flux N_{eff} at fixed r decreases with time
→ effective 't Hooft coupling N_{eff}/k decreases

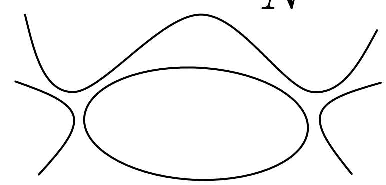
The O(N) vector model

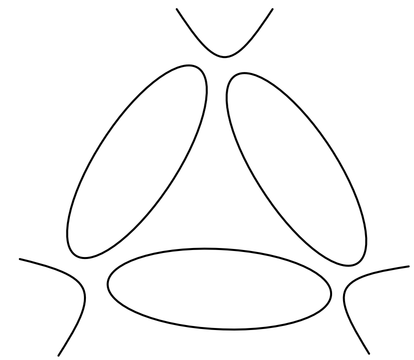
$$S = \int d^3x \left(-\partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} - \frac{\lambda}{6N^2} \left(\vec{\phi} \cdot \vec{\phi} \right)^3 \right)$$



$$\beta_{\mathrm{pert}}(\lambda) = \frac{3}{2\pi^2 N} \left(\lambda^2 - \frac{\lambda^3}{192} \right) + \text{ higher order in } \frac{1}{N}$$

Pisarski





Negative coupling $(\lambda < 0)$:

- Perturbative UV fixed point at $\lambda = 0$ → asymptotically free

Coleman, Gross

Positive coupling $(\lambda > 0)$:

- Perturbative UV fixed point: $\lambda^* = 192$
- Non-perturbative instability at leading order in 1/N for $\lambda > \lambda_c = 16\pi^2$
  Can show: $V_{eff} \to -\infty$ as $\langle \phi^2 \rangle_{\mathrm{ren}} \to -\infty$
  (With cutoff: all masses of order the cutoff)

Bardeen, Moshe, Bander

- Recent work: study time-dependent states: $\langle \phi^2 \rangle_{\mathrm{ren}} = -\frac{CN}{t}$

Asnin, Rabinovici, Smolkin; BC, Hertog, Turok

The O(N) vector model: summary

$$S = \int d^3x \left(-\partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} - \frac{\lambda}{6N^2} \left(\vec{\phi} \cdot \vec{\phi} \right)^3 \right)$$

- classical instability for $\lambda < 0$: $\quad \phi \sim \frac{1}{\sqrt{|t|}}$
- quantum instability for $\lambda > \lambda_c$: $\quad \langle \phi^2 \rangle \sim \frac{1}{|t|}$
- perturbative fixed point in quantum unstable regime

The O(N) x O(N) vector model

$$S = \int d^3x \left[-\partial_\mu \vec{\phi}_1 \cdot \partial^\mu \vec{\phi}_1 - \partial_\mu \vec{\phi}_2 \cdot \partial^\mu \vec{\phi}_2 - \frac{\lambda_{111}}{6N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 \right)^3 - \frac{\lambda_{222}}{6N^2} \left(\vec{\phi}_2 \cdot \vec{\phi}_2 \right)^3 - \frac{\lambda_{112}}{6N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 \right)^2 \left(\vec{\phi}_2 \cdot \vec{\phi}_2 \right) - \frac{\lambda_{122}}{6N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 \right) \left(\vec{\phi}_2 \cdot \vec{\phi}_2 \right)^2 \right]$$

Rabinovici, Saering, Bardeen

Special case: $V = \frac{\lambda}{6N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 - \vec{\phi}_2 \cdot \vec{\phi}_2 \right)^3 \qquad (\lambda < 0)$

Results:

- flows to UV fixed point under perturbative RG flow: $\begin{cases} \lambda_{222} = \lambda^* \\ \lambda_{112} = \lambda_{122} = \lambda_{111} = 0 \end{cases}$
- computed approach of fixed point
- however, quantum instability will kick in before λ_{222} reaches λ^*

BC, Hertog, Turok

The O(N) x O(N) vector model: time evolution

$$S = \int d^3x \left[-\partial_\mu \vec{\phi}_1 \cdot \partial^\mu \vec{\phi}_1 - \partial_\mu \vec{\phi}_2 \cdot \partial^\mu \vec{\phi}_2 - \frac{\lambda_{111}}{6N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 \right)^3 - \frac{\lambda_{222}}{6N^2} \left(\vec{\phi}_2 \cdot \vec{\phi}_2 \right)^3 - \frac{\lambda_{112}}{6N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 \right)^2 \left(\vec{\phi}_2 \cdot \vec{\phi}_2 \right) - \frac{\lambda_{122}}{6N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 \right) \left(\vec{\phi}_2 \cdot \vec{\phi}_2 \right)^2 \right]$$

Rabinovici, Saering, Bardeen

Special case: $V = \frac{\lambda}{6N^2} \left(\vec{\phi}_1 \cdot \vec{\phi}_1 - \vec{\phi}_2 \cdot \vec{\phi}_2 \right)^3 \qquad (\lambda < 0)$

ϕ_1 classically rolls to large values → UV → $\lambda_{222} > \lambda_c$

→ ϕ_2 quantum unstable → coupled system

BC, Hertog, Turok (work in progress)

Summary

- Cosmological models are incomplete without an understanding of the cosmological singularity.
- The AdS/CFT correspondence relates gravitational theories allowing big crunch singularities to field theories with potentials unbounded from below.
- Bulk and boundary instabilities (and their relation) can be understood in terms of expanding spherical branes.
- We need to understand dynamics with unbounded potentials. We have seen preliminary results in two concrete models.