Non-linear supersymmetry and brane dynamics

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1. Introduction and motivations
2. $N = 1$: goldstino couplings to matter

   I.A.-Tuckmantel '04; Komargodski-Seiberg '09

   Non-linear MSSM — I.A.-Dudas-Ghilencea-Tziveloglou to appear
3. Extended supersymmetry and brane dynamics

   Bagger-Galperin '97; I.A.-Derendinger-Maillard '08

   Ambrosetti-I.A.-Derendinger-Tziveloglou '09 + to appear

Non-linear supersymmetry ⇒ goldstino mode χ

Volkov-Akulov '73

Why study goldstino interactions:

- Effective field theory of SUSY breaking at low energies $m_\chi << m_{susy}$

  e.g. gauge mediation dominant vs gravity mediation

  χ: longitudinal gravitino with $m_\chi << m_{soft} << m_{susy}$

- Brane dynamics: half SUSY of the bulk broken but NL realized

  ⇒ e.g. strongly constrain coupling of brane to bulk fields

  exact NL susy in the large volume limit

  broken by the orientifold projection at finite volume

  ⇒ important for large volume compactifications, e.g. low scale strings

Non-linear SUSY transformations: [5]

$$\delta\chi_\alpha = \frac{\xi_\alpha}{\kappa} + \kappa\,\Lambda^\mu_\xi\,\partial_\mu\chi_\alpha \qquad \Lambda^\mu_\xi = -i\left(\chi\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\chi}\right)$$

κ: goldstino decay constant (SUSY breaking scale) $\kappa = (\sqrt{2}m_{susy})^{-2}$

Goldstino interactions: 3 formulations

- Standard realization

  Volkov-Akulov '73, Clark-Love '96, Clark-Lee-Love-Wu '98

- Superfield formalism Ivanov-Kapustnikov '78, Samuel-Wess '83

  Brignole-Feruglio-Zwirner '97, Luty-Ponton '98, I.A.-Tuckmantel '04

- Constrained superfields

  Rocek-Tseytlin '78, Lindstrom-Rocek '79, Komargodski-Seiberg '09

Standard realization

Define the 'metric': $G_\mu^\nu = \delta_\mu^\nu + \kappa^2\, t_\mu^\nu \qquad t_\mu^\nu = i\chi \overset{\leftrightarrow}{\partial}_\mu \sigma^\nu \bar{\chi}$

$\delta(\det G) = \kappa\, \partial_\mu \left(\Lambda_\xi^\mu \det G\right) \Rightarrow$ invariant action:

$$S_{VA} = -\tfrac{1}{2\kappa^2}\int d^4x \det G = -\tfrac{1}{2\kappa^2} - \tfrac{i}{2}\chi\sigma^\mu \overset{\leftrightarrow}{\partial}_\mu \bar{\chi} + \dots$$

Generalization to matter and gauge fields:

$$S_{eff} = \int d^4x \det G\, \mathcal{L}_{SM}(\phi)$$ invariant if $\delta\phi = \kappa\, \Lambda_\xi^\mu \partial_\mu \phi$ and so $\mathcal{L}_{SM}$

However problem with derivatives ⇒ define SUSY covariant ones:

$$\mathcal{D}_\mu\phi \equiv \left(G^{-1}\right)_\mu^\nu D_\nu\phi \qquad \mathcal{F}_{\mu\nu} \equiv \left(G^{-1}\right)_\mu^\lambda \left(G^{-1}\right)_\mu^\rho F_{\lambda\rho}$$

$$\mathcal{L}_{eff} = \det G\, \mathcal{L}_{SM}(\phi, \mathcal{D}_\mu\phi) = \mathcal{L}_{SM}(\phi, D_\mu\phi) + \kappa^2\, t^{\mu\nu}\, T_{\mu\nu} + \dots$$

universal coupling to stress-tensor but NOT the most general inv action

Superfield formalism

Recipe: $\phi(x) \rightarrow \Phi(x,\theta,\bar{\theta}) \equiv \phi(\tilde{x})$ $\quad \tilde{x}^\mu = x^\mu + \Lambda^\mu_\theta(\tilde{x})$ [3] [8]

$$= \phi(x) + \kappa\Lambda^\mu_\theta \partial_\mu \phi + \ldots \quad \Rightarrow$$

Goldstino (spinor) superfield: $\mathcal{G}_\alpha = \frac{\theta_\alpha}{\kappa} + \chi_\alpha(\tilde{x})$

space-time derivatives: use the 'metric' $G(\tilde{x})$

$$\text{e.g. } \mathcal{F}_{\mu\nu}(x,\theta,\bar{\theta}) \equiv \left[\left(G^{-1}\right)^\lambda_\mu \left(G^{-1}\right)^\rho_\mu F_{\lambda\rho} \right](\tilde{x})$$

$$O = \int d^2\theta d^2\bar{\theta}\,\mathcal{O} = \sum_{n\geq 0} \kappa^n O^{(n)}$$ even/odd $n \leftrightarrow$ even/odd number of χ's

dims: $[\mathcal{O}] = d \geq 0 \Rightarrow [O] = d + 2\ ,\ [O^{(n)}] = d + 2 + 2n$

Effective operators of dimension $\leq 8 \Rightarrow\ d \leq 2\,,\ n \leq 2$

List of lowest dim operators

2 operators of dim 6 linear in χ [12]

$$S_1 = C_1 \int d^4x \; \kappa \, F_{\mu\nu} \psi \sigma^\mu \partial^\nu \bar{\chi} + h.c. \qquad S_2 = C_2 \int d^4x \; \kappa \, (\psi \partial_\alpha \chi) D^\alpha \phi + h.c.$$

Quadratic in χ: 1 operator of dim 7

$$S_7 = C_7 \int d^4x \; \kappa^{3/2} \phi_1 \phi_2 \; \partial_\mu \chi J^{\mu\nu}_{(\frac{1}{2},0)} \partial_\nu \chi + h.c. \qquad J^{\mu\nu}_{(\frac{1}{2},0)} = \frac{i}{4} \sigma^{[\mu} \bar{\sigma}^{\nu]}$$

$+$ 5 operators of dim 8

$$S_3 = C_3 \int d^4x \, \kappa^2 (\psi_1 \partial^\mu \chi)(\bar{\psi}_2 \partial_\mu \bar{\chi}) + h.c. \quad S_4 = C_4 \int d^4x \, \kappa^2 (\psi_1 \psi_2)(\partial_\mu \chi \partial^\mu \chi) + h.c.$$

$$S_5 = C_5 \int d^4x \, \kappa^2 \phi_1 \overleftrightarrow{D}_\mu \phi_2 i \partial_\alpha \chi \sigma^\mu \partial^\alpha \bar{\chi} + h.c.$$

$$S_6 = C_6 \int d^4x \, \kappa^2 \partial^\alpha \chi \sigma^\mu \partial^\nu \bar{\chi} \partial_\alpha F_{\mu\nu} + h.c.$$

$$S_8 = C_8 \int d^4x \, \kappa^2 \phi_1 \phi_2 \phi_3 (\partial_\mu \chi J^{\mu\nu}_{(\frac{1}{2},0)} \partial_\nu \chi) + h.c.$$

Constrained superfields

spontaneous global $S\!\!\!/USY$: no supercharge but still conserved supercurrent

$\Rightarrow$ superpartners exist in operator space (not as 1-particle states)

$\Rightarrow$ constrained superfields: 'eliminate' superpartners

Goldstino: chiral superfield X_{NL} satisfying $X_{NL}^2 = 0 \Rightarrow$ [19]

$$X_{NL}(y) = \frac{\chi^2}{2F} + \sqrt{2}\theta\chi + \theta^2 F \qquad y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$$

$$= F\Theta^2 \qquad \Theta = \theta + \frac{\chi}{\sqrt{2}F}$$

$$\mathcal{L}_{NL} = \int d^4\theta X_{NL}\bar{X}_{NL} - \frac{1}{\sqrt{2}\kappa}\left\{\int d^2\theta X_{NL} + h.c.\right\} = \mathcal{L}_{VA}$$

$$F = \frac{1}{\sqrt{2}\kappa} + \ldots$$

Goldstino couplings to matter

Coupling to superfields: $m_{soft} << E << m_{susy} \sim 1/\sqrt{\kappa}$

replace auxiliary superfield spyrion $S = m_{soft}\theta^2$ by $\sqrt{2}\kappa m_{soft} X_{NL}$

Coupling to (non-SUSY) matter: $E << m_{soft}, m_{susy}$

→ constrained matter superfelds

- Fermions: Q_{NL} satisfying $Q_{NL}X_{NL} = 0$ (eliminate sfermions) $\Rightarrow$

$$Q_{NL} = \sqrt{2}\left(\psi - \frac{F_Q\chi}{F}\right)\Theta + F_Q\Theta^2$$

- Complex scalars: H_{NL} with $X_{NL}\bar{H}_{NL} =$ chiral (eliminate 'higgsinos') [5]

$$\Rightarrow H_{NL} = H(\hat{y}) \qquad \hat{y} = y^\mu + i\sqrt{2}\theta\sigma^\mu\bar{\chi}(\hat{y})/\bar{F}(\hat{y})$$

- Real scalars: A_{NL} with $X_{NL}\bar{A}_{NL} = X_{NL}A_{NL}$

$$\Rightarrow \mathrm{Im}A = \tfrac{1}{2F\bar{F}}\left(\chi\sigma^\mu\bar{\chi}\right)\partial_\mu \mathrm{Re}A + \ldots$$

- Gauge fields: V_{NL}

  gauge transformations: $\delta V = \Omega + \bar{\Omega}$ with $X_{NL}(\Omega_{NL} + \bar{\Omega}_{NL}) = 0$

  e.g. charged matter $H_{NL} = e^{iR_{NL}}$; $X_{NL}(R_{NL} - \bar{R}_{NL}) = 0$: $\delta R_{NL} = i\Omega_{NL}$

  $\Rightarrow$ convenient gauge choice $X_{NL} V_{NL} = 0$

  eliminate gaugino: $X_{NL} W_{NL} = 0$ field strength $W = -\frac{1}{4}\bar{D}^2 DV$ [19]

  $$\Rightarrow V_{NL} = -\Theta\left(\sigma^m V_m + \frac{D}{|F|^2}\chi\bar{\chi}\right)\bar{\Theta} + \frac{1}{2}\Theta^2\bar{\Theta}^2 D + \text{derivatives}$$

D-brane dynamics

Type II (closed) strings on $4d$ Minkowski $M_4 \times X_6$ internal $6d$ manifold

X_6 flat $\Rightarrow N = 8$ SUSY ; X_6 Calabi-Yau $\Rightarrow N = 2$ SUSY

Single stack of N Dp-branes $\Rightarrow$ half SUSY is spontaneously broken $p \geq 3$

$(p-3)$ dims wrapped around cycles in $X_6 \Rightarrow 4d$ effective field theory

- Gauge group: $G = U(N)$ (generically)
- SUSY: half remains unbroken Q_e ; other half NL realized Q_o

  broken SUSY commutes with $G \Rightarrow$ goldstino $= U(1)$ gaugino of Q_e

Intersecting branes: useful framework for model building

Standard Model embedding

Two D-brane stacks: N_1 Dp_1 and N_2 Dp_2

$\Rightarrow$ bifundamental matter on their intersections: chiral fermions

L-SUSY: generally broken but preserved for special intersection angles

e.g. for $X_6 = T^2 \times T^2 \times T^2$ when $\theta_1 + \theta_2 + \theta_3 = 0$

NL-SUSY: generally all (both $U(1)_1 \times U(1)_2$ gauginos = goldstinos)

special angles $\Rightarrow$ only a linear combination

Remark: string consistency (e.g. tadpole cancellation) $\Rightarrow$ need orientifolds

non-dynamical planes $\Rightarrow$ break half-SUSY explicitly

$\Rightarrow$ goldstino gets a volume suppressed mass

NL-SUSY only locally → restored in the large volume limit

Intersecting D-brane models

I.A.-Tuckmantel '04

1) Goldstino decay constant: sum of brane tensions

$$\frac{1}{2\kappa^2} = T_1 + T_2 \qquad T_i = \frac{M_s^4}{4\pi^2 g_i^2} N_i$$

2) Goldstino couplings: only 3 non-vanishing up to order κ^2 [6]

$$C_1 = \sqrt{2} \quad ; \quad C_2 = 2 \quad ; \quad C_3 = 2$$

- universal coefficients independent of brane-angles
- C_3: fixes the field theory ambiguity of 4-fermion operator

Brignole-Feruglio-Zwirner '97, I.A.-Benakli-Laugier '01

- $C_{1,2}$: dim 6 operators linear in χ can be written as

$\mathcal{L}_{\rm linear} = \frac{\kappa}{\sqrt{2}} J^\mu \partial_\mu \chi + h.c.$ J^μ : $N = 1$ supercurrent of linear SUSY

present in the intersection if massless scalars

Phenomenological analysis in the Standard Model

$$\mathcal{L}_\chi = -\frac{i}{2}\chi\sigma^\mu\overset{\leftrightarrow}{\partial}_\mu\bar{\chi} + i\kappa^2(\chi\overset{\leftrightarrow}{\partial^\mu}\sigma^\nu\bar{\chi})T_{\mu\nu} + \delta\mathcal{L}_\chi$$

$$\delta\mathcal{L}_\chi = i\sqrt{2}\kappa F_{\mu\nu}\psi\sigma^\mu\partial^\nu\bar{\chi} + 2\kappa D_\mu\phi(\psi\partial^\mu\chi) + h.c.$$
$$+2\kappa^2(\partial_\mu\chi\psi_1)(\partial^\mu\bar{\chi}\bar{\psi}_2) + \mathcal{O}(\kappa^3)$$

- 1st term: hypercharge + fermion singlet
- 2nd term: higgs + lepton doublets I.A.-Tuckmantel-Zwirner '04

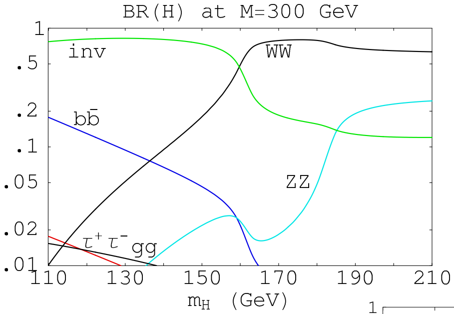
  preserves lepton number if $L(\chi) = -1$

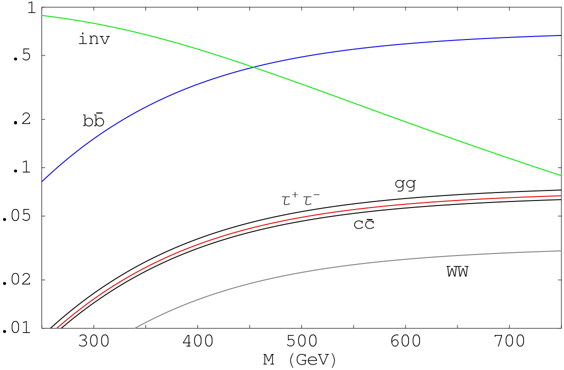
  $Z, H \to \nu\chi$ $W^\pm \to l^\pm\chi \Rightarrow$ $M = m_{susy}\,|\sqrt{2}/C_2|^{1/2} \simeq M_s/2$

  - bounds: $M \gtrsim 270$ GeV $\Rightarrow M_s \gtrsim 500$ GeV (e.g. invisible Z width)

  - signal: invisible Higgs decay

    dominant or non-negligible in a large range of (M, m_H)


BR(H) at M=300 GeV
1
.5
.2
.1
.05
.02
.01
inv
WW
$b\bar{b}$
ZZ
$\tau^+\tau^-$
gg
110
130
150
170
190
210
m_H (GeV)


BR(H) at m_H = 115 GeV
1
.5
.2
.1
.05
.02
.01
inv
$b\bar{b}$
gg
$\tau^+\tau^-$
$c\bar{c}$
WW
300
400
500
600
700
M (GeV)

Phenomenological analysis in the MSSM

$E \sim m_{soft} >> m_\chi$ **I.A.-Dudas-Ghilencea-Tziveloglou to appear**

Higgs potential is modified:

$$V = V_{MSSM} + 2\kappa^2 \left| m_1^2 |h_1|^2 + m_2^2 |h_2|^2 + B\mu h_1 h_2 \right|^2 + \mathcal{O}(\kappa^4) \quad \Rightarrow$$

$m_{1,2}, B\mu$: soft mass parameters, μ: higgsino mass

classical value of light higgs mass can be increased above the LEP bound

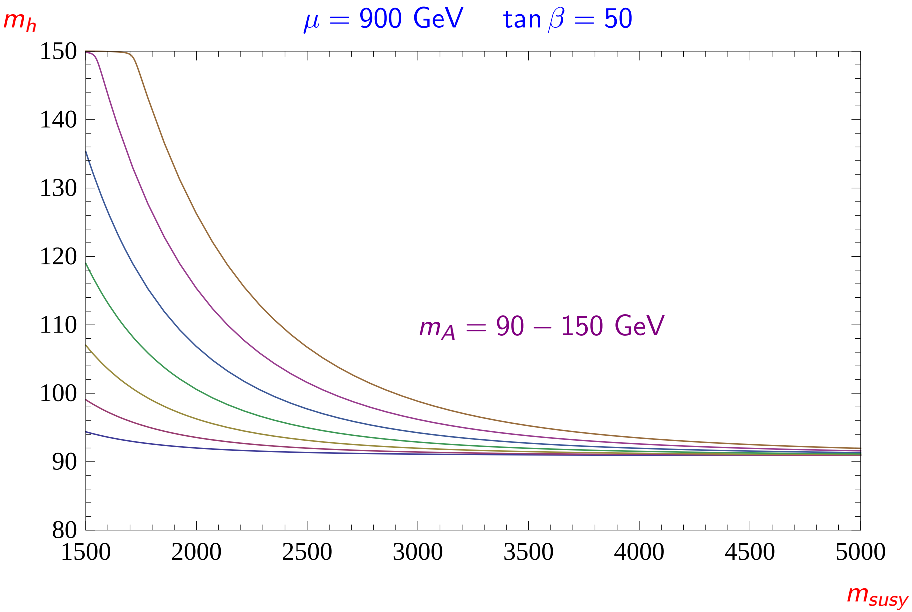
large $\tan\beta$ limit: $m_h^2 = m_Z^2 + \frac{v^2}{2m_{susy}^2}(2\mu^2 + m_Z^2)^2 + \cdots$

→ e.g. $\mu = 900$ GeV, $m_{susy} = 2$ TeV $\Rightarrow$ $m_h = 114.4$ GeV

Quartic higgs coupling increases for large soft masses ⇒

MSSM 'little' fine tuning is alleviated

New couplings ⇒ $Z, h \to \chi\chi$ ⇒ invisible higgs decay


$\mu = 900$ GeV $\tan\beta = 50$
m_h
150
140
130
120
110
100
90
80
1500
2000
2500
3000
3500
4000
4500
5000
$m_A = 90 - 150$ GeV
m_{susy}

NL extended supersymmetry $N = 1_L + 1_{NL}$

Goldstino in multiplet of $N = 1$ SUSY: vector or chiral?

brane dynamics ⇒ Maxwell goldstino multiplet

gauge chiral multiplet$\big|_{N=2}\, \mathcal{W} = (\text{vector } W + \text{chiral } X)_{N=1}$

$$\mathcal{W}(y,\theta,\tilde\theta) = X(y,\theta) + i\sqrt{2}\tilde\theta W(y,\theta) - \tilde\theta^2 \left[\tfrac{1}{4}\overline{DDX}(y,\theta) + \tfrac{1}{2\kappa}\right]$$

allow partial SUSY breaking $N = 2 \rightarrow N = 1$ ↗

$\delta^* X = i\sqrt{2}\eta^\alpha W_\alpha \qquad \delta^* W_\alpha = \frac{i}{\sqrt{2}\kappa}\eta_\alpha + \dots$ ← linear SUSY

$$\mathcal{L}^{N=2}_{Maxwell} = -\frac{1}{8}\int d^2\theta\, d^2\tilde\theta\, \mathcal{W}^2 + h.c. = \int d^2\theta \left[\frac{1}{2}W^2 - \frac{1}{4}X\overline{DDX} - \frac{1}{2\kappa}X\right] + h.c.$$

Partial SUSY breaking: non trivial prepotential $f(\mathcal{W})$ [19]

I.A.-Partouche-Taylor '96

Partial SUSY breaking

$$\begin{aligned}\mathcal{L}_{partial} &= -\frac{1}{8}\int d^2\theta\, d^2\tilde{\theta}\, f(\mathcal{W}) - \frac{1}{8}\xi_1\int d^2\theta\, X + h.c.\\ &= \frac{1}{4}\int d^2\theta\left[f''(X)W^2 - \frac{1}{2}f'(X)\overline{DDX} \underbrace{-\frac{1}{\kappa}}_{\nearrow} f'(X) - \frac{1}{2}\xi_1 X\right] + h.c.\end{aligned}$$

magnetic FI term

⇒ scalar potential: $V_{scalar} = \frac{1}{16\,\mathrm{Re}f''}\left|\frac{1}{2}\xi_1 + \frac{1}{\kappa}f''\right|^2$

non-trivial prepotential f ⇒ partial SUSY breaking for $f''(X) = -\kappa\,\xi_1/2$

Non-linear $N = 2$ constraint: $\mathcal{W}_{NL}^2 = 0$

$$\Rightarrow \quad X^2 = 0 \quad , \quad XW_\alpha = 0 \quad , \quad WW - \tfrac{1}{2} X\overline{DDX} = \tfrac{1}{\kappa} X$$ [7]

$$X = \kappa W^2 - \kappa^3 \bar{D}^2 \frac{W^2 \overline{W}^2}{1+A_+ + \sqrt{1+2A_+ + A_-^2}} \qquad A_\pm = \tfrac{\kappa^2}{2} \left(D^2 W^2 \pm \bar{D}^2 \overline{W}^2 \right) = \pm A_\pm^*$$

$$\Rightarrow \mathcal{L}_{NL}^{N=2} = \tfrac{1}{4\kappa} \int d^2\theta X + h.c.$$

$$= \tfrac{1}{8\kappa^2} \left(1 - \sqrt{-\det(\eta_{\mu\nu} + 2\sqrt{2}\kappa F_{\mu\nu})} \right) + \ldots = \mathcal{L}_{\rm DBI} \leftarrow \text{D-brane}$$

The FI-term is also invariant under NL SUSY

$$\mathcal{L}_{FI} = \xi \int d^4\theta V; \quad W = -\tfrac{1}{4} \bar{D}^2 DV; \quad \delta^* V = \tfrac{i}{2\kappa} \left(\eta D + \bar{\eta} \bar{D} \right) \theta^2 \bar{\theta}^2 + \ldots$$

$$\Rightarrow \mathcal{L}_{Max}^{NL} = \mathcal{L}_{NL}^{N=2} + \mathcal{L}_{FI}$$ [21]

Coupling to bulk hypermultiplets e.g. the universal dilaton

at least one isometry → single-tensor multiplet (RR 2-forms)

$N = 2$ tensor $\mathcal{Y} = (\text{tensor } L + \text{chiral } \Phi)_{N=1}$ $\qquad D^2 L = \bar{D}^2 L = 0$

general action: $\mathcal{L}_{ST} = \int d^4\theta \mathcal{H}(L, \Phi, \bar{\Phi})$ with $\left(\partial_L^2 + 2\partial_\Phi \partial_{\bar{\Phi}}\right) \mathcal{H} = 0$ [24]

$L = D^\alpha \ell_\alpha + h.c.$ $\qquad \ell_\alpha$: chiral spinor superfield

off-shell $N = 2 \Rightarrow$ add auxiliary chiral superfield $Y \sim \theta^2 \epsilon \cdot C_4 \leftarrow$ 4-form

$$\mathcal{Y}(y, \theta, \tilde{\theta}) = Y(y, \theta) + i\sqrt{2}\, \tilde{\theta}\, \ell(y, \theta) - \tfrac{i}{2} \tilde{\theta}^2\, \Phi(y, \theta)$$

coupling to $N = 2$ vector $\mathcal{W}$: Chern-Simons interaction:

$$\mathcal{L}_{CS} \sim g \int d^2\theta d^2\tilde{\theta}\, \mathcal{Y}\, \mathcal{W} = g \int d^2\theta \left(\ell^\alpha W_\alpha + \frac{1}{2} \Phi X - \frac{i}{2\kappa} Y \right) + h.c.$$

⇒ global SUSY limit of D-brane coupling to bulk hypermultiplets

$N=2$ NL QED and novel super-higgs mechanism

General action: $\mathcal{L}_{tot}^{NL} = \mathcal{L}_{CS} + \mathcal{L}_{Max}^{NL}(W) + \mathcal{L}_{ST}(L,\Phi)$ $\Rrightarrow$ [19]

superhiggs mechanism without gravity:

Maxwell goldstino $\mathcal{W}_{NL}(W)$ is ‘absorbed’ by $N=2$ tensor $\mathcal{Y}(L,\Phi)$

$\rightarrow$ $N=1$ massive vector (W,L) + massless chiral Φ:

- tensor of L + vector of W $\rightarrow$ massive vector
- scalar of $L \in$ same massive vector multiplet
- goldstino + fermion of L $\rightarrow$ Dirac spinor

System identical to Higgs phase of $N=2$ NL QED (up to $\mathcal{L}_{ST}$)

$\Phi \sim Q_1 Q_2 \qquad L \sim |Q_1|^2 - |Q_2|^2 \qquad (Q_1, Q_2)$: charged hypermultiplet

adding mass-term $m\int d^2\theta\, \Phi \Rrightarrow$ also Coulomb phase for $\xi = 0$

Vacuum structure of $N = 2$ NL QED

3 parameters: κ, ξ, m

- $m = 0$: Higgs phase and super-higgs without gravity
  $\langle Q_1 \rangle = v$ (real) arbitrary $\qquad \langle Q_2 \rangle = \sqrt{\xi + v^2}$
- $m \neq 0, \xi = 0$: Coulomb phase with $N = 2$ NL SUSY unbroken
- $m \neq 0, \xi \neq 0$: $N = 1$ (linear) SUSY is also broken

Ambrosetti-I.A.-Derendinger-Tziveloglou '09

global limit of the universal hypermultiplet

Ambrosetti-I.A.-Derendinger-Tziveloglou '10

type IIB string basis:

NS-NS sector: dilaton φ + (2-form $B_{\mu\nu} \leftrightarrow$ 4d axion a)

R-R sector: scalar $C^{(0)}$ + 2-form $C_{\mu\nu} \leftrightarrow$ complex scalar C

perturbation theory: 3 isometries (a, C shifts) forming the Heiseberg group

$[T_1, T_2] = T_a \qquad [T_{1,2}, T_a] = 0$

extended by a 4th generator R rotating (T_1, T_2) : phase transform of C

$[R, T_i] = \varepsilon_{ij} T_j \qquad [R, T_a] = 0 \qquad \Rightarrow$

T_a = central extension of the 2-dim Euclidean group E_2: (T_1, T_2, R)

$N = 2$ **local SUSY**: 4d quaternionic manifold $\equiv$

(Weyl) self-dual Einstein space of non-zero curvature

Heiseberg isometry $\Rightarrow$ potential $F(\rho)$ of one variable $\rho \leftrightarrow$ dilaton:

$$F = \rho^{3/2} + \chi\rho^{-1/2}$$

8 isometries $SU(2,1)/U(2)$ ← tree-level   1-loop correction

Caldebank-Pedersen '01, I.A.-Minasian-Theisen-Vanhove '03

$N = 2$ **global SUSY**: 4d hyperkähler manifold $\equiv$

(Riemann) self-dual Einstein space of zero curvature

Heiseberg isometry $\Rightarrow$ one variable: [20]

$$\mathcal{V} = L + \frac{i}{\sqrt{2}}(\Phi + \bar{\Phi}) \qquad \mathcal{H}(\mathcal{V}) = -\frac{A}{6}\mathcal{V}^3 - B\mathcal{V}^2 + h.c.$$

Zero-curvature limit $\kappa_N \to 0$: rescale appropriately all scalar fields C, a

and the dilaton around a fixed constant value

$$\kappa_N^2 ds^2_{CPH} = \frac{\rho^2-\chi}{(\rho^2+\chi)^2}(d\rho^2+dC_1^2+dC_2^2)+\frac{4\rho^2}{(\rho^2+\chi)^2(\rho^2-\chi)}(da+C_1\,dC_2)^2$$

$$\rho^2 = e_4^{-2\phi}-\chi \qquad \chi=\chi_E/12\pi$$

$$ds^2_{global} = \frac{A\ell+B}{2}\left[d\ell^2+2\left(d\Phi_1^2+d\Phi_2^2\right)\right]+\frac{1}{2(A\ell+B)}(db+2A\,\Phi_1\,d\Phi_2)^2$$

Limit CPH → global:

$$a=\chi\,\kappa_N^{4/3}\mu^{1/3}\,b \qquad C_{1,2}=\sqrt{2\chi}\,\kappa_N^{2/3}\mu^{-1/3}\Phi_{1,2}$$

$e^{-2\phi_4}=2\chi\,(1+\kappa_N^{2/3}\mu^{-1/3}\,\ell) \quad \Rightarrow \quad$ 4d string coupling: $e^{\langle\phi_4\rangle}=\frac{1}{\sqrt{2\chi}}$

requires $\chi_E>0$

Conclusions

Non-linear supersymmetry: powerful tool for studying:

- low energy SUSY breaking $E << m_{SUSY} \sim 1/\sqrt{\kappa}$

  Volkov-Akulov action and goldstino χ couplings to matter

  standard coupling to stress-tensor *not* the most general

  $\rightarrow$ detailed analysis $\Rrightarrow$ dim 6 operators linear in χ

  $E >> m_{soft} \Rrightarrow$ goldstino $\equiv$ spurion coupled to supermultiplets

  $E << m_{soft} \Rrightarrow$ goldstino coupling to Standard Model fields

- brane effective actions $\Rrightarrow$ brane dynamics

  $N = 1$ string computation: 3 independent couplings up to dim 8

  $N = 2$ NL SUSY $\Rrightarrow$ DBI action and couplings to bulk fields

vacuum structure of NL QED and superhiggs without gravity