Anomalies and hydrodynamics

Amos Yarom

(Together with K. Jensen, R. Loganayagam)

Anomalies

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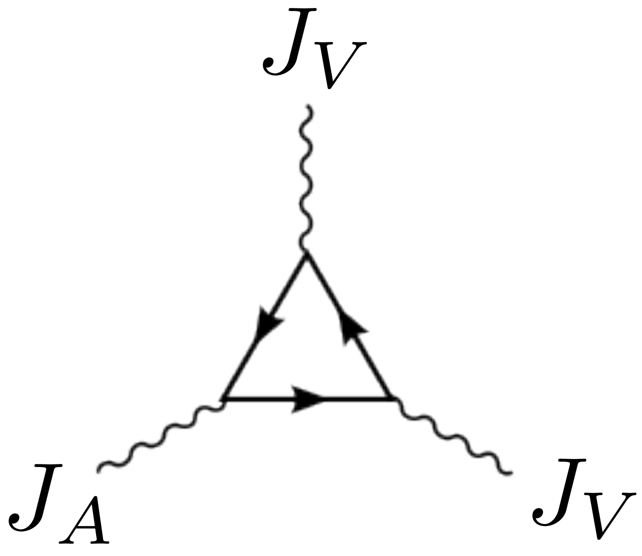

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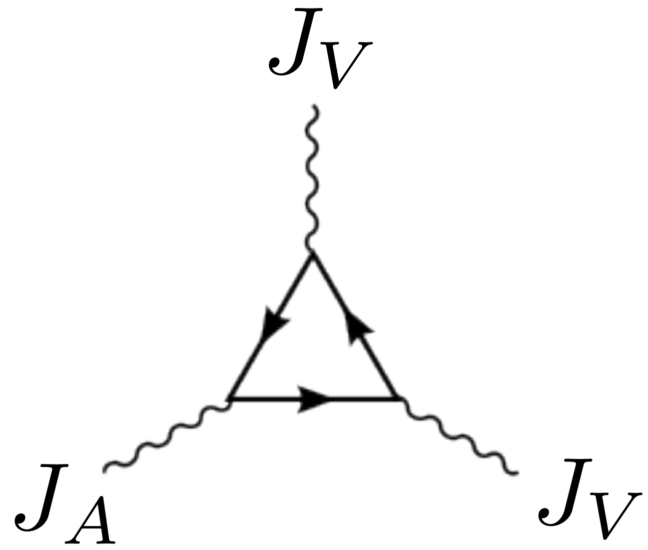

$$\partial_\mu J^\mu_A = -\frac{e^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\sigma\lambda}$$

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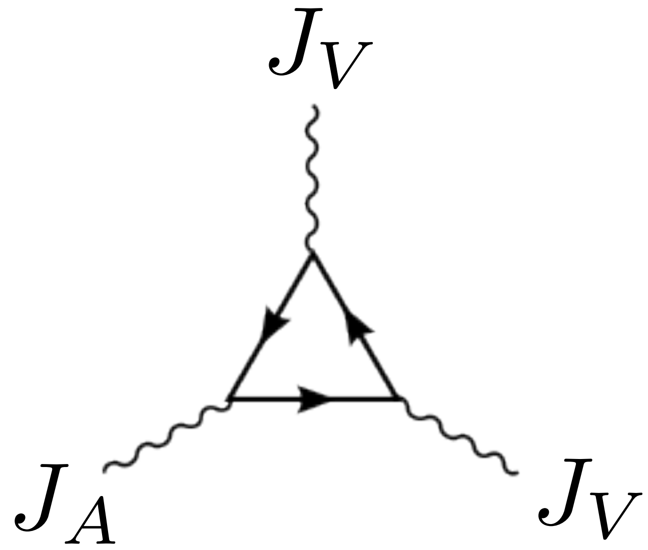

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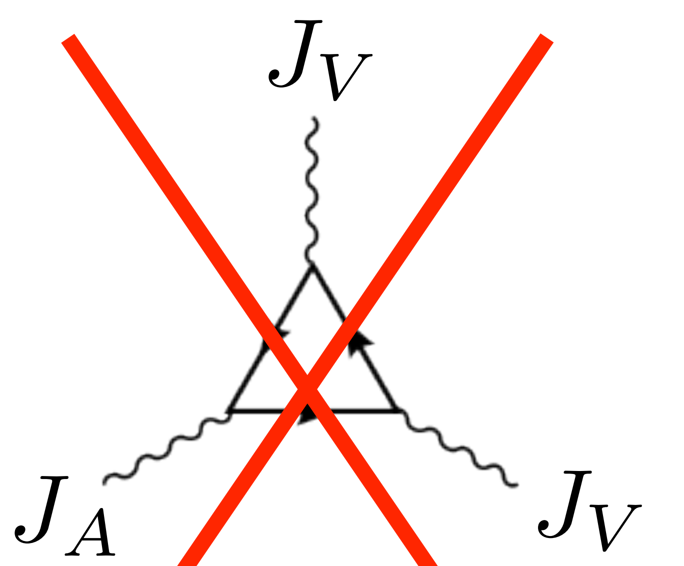

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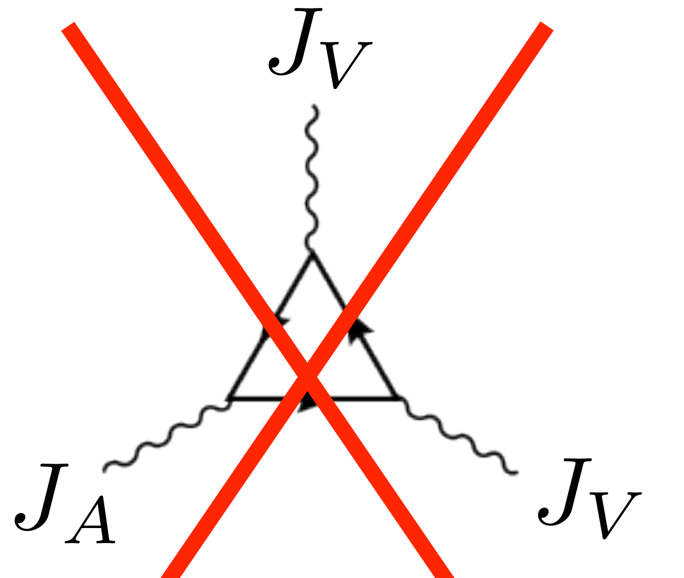

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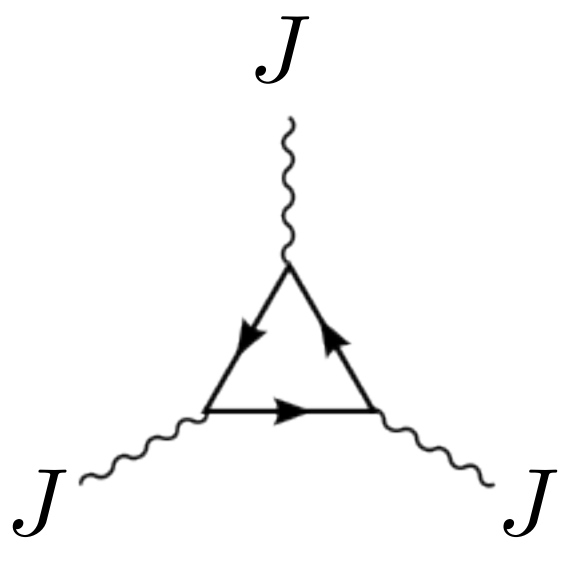

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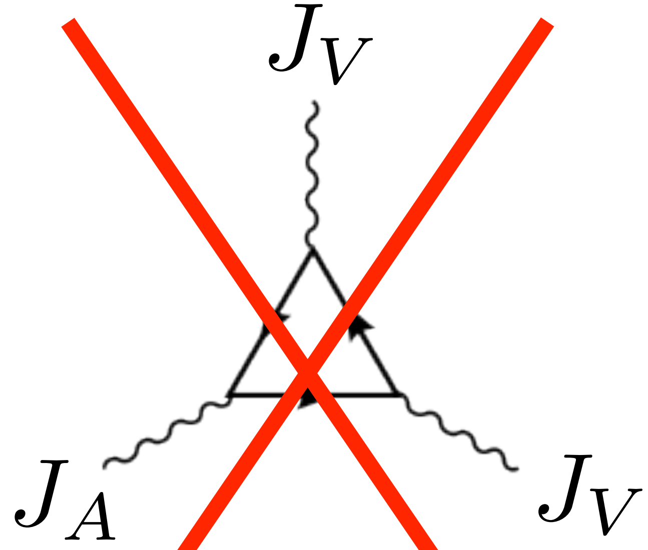

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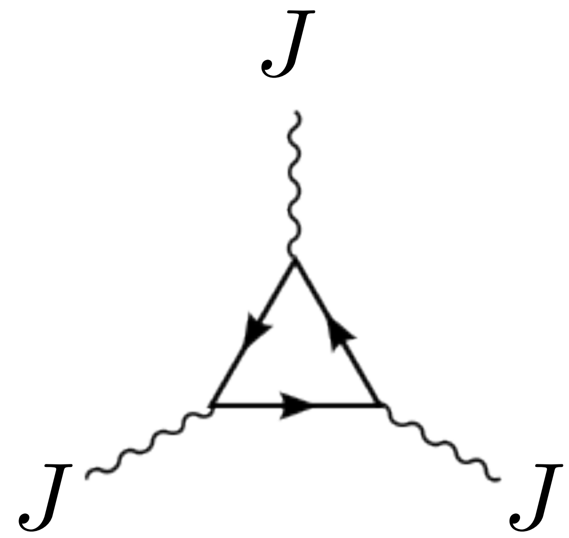

$$\partial_\mu J^\mu = \frac{3}{4}\epsilon^{\mu\nu\rho\sigma}c_A F_{\mu\nu}F_{\rho\sigma}$$

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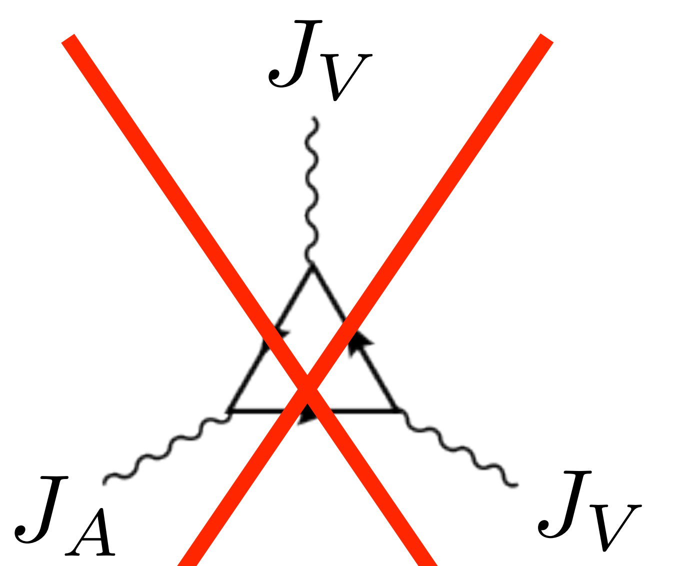

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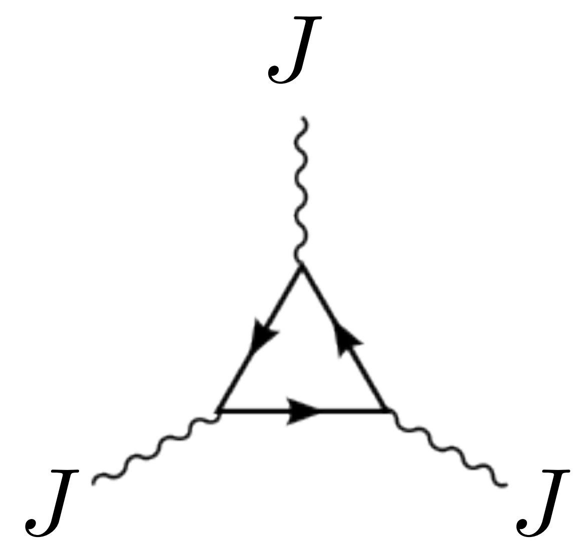

$$\partial_\mu J^\mu = \frac{3}{4}\epsilon^{\mu\nu\rho\sigma}c_A F_{\mu\nu}F_{\rho\sigma}$$

$$c_A = -\frac{2\pi}{3!(2\pi)^3}\sum_{i=species}\chi_i(q_i)^3$$

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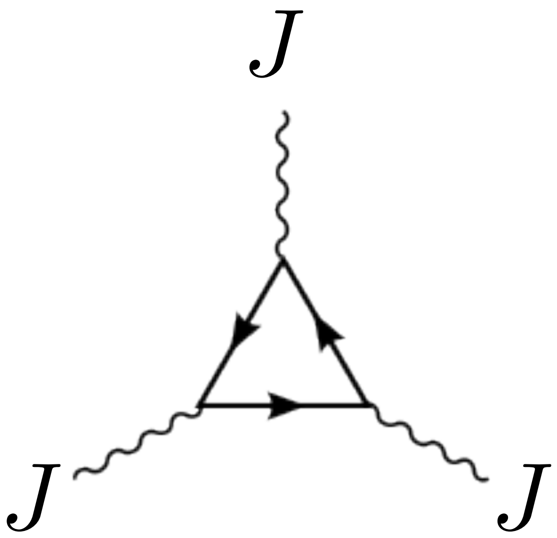

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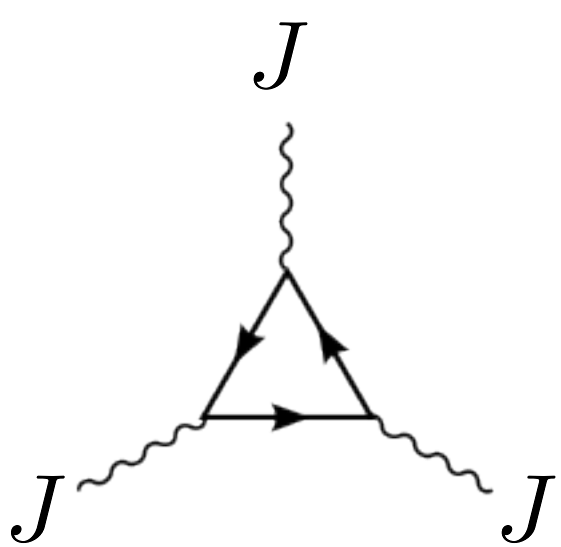

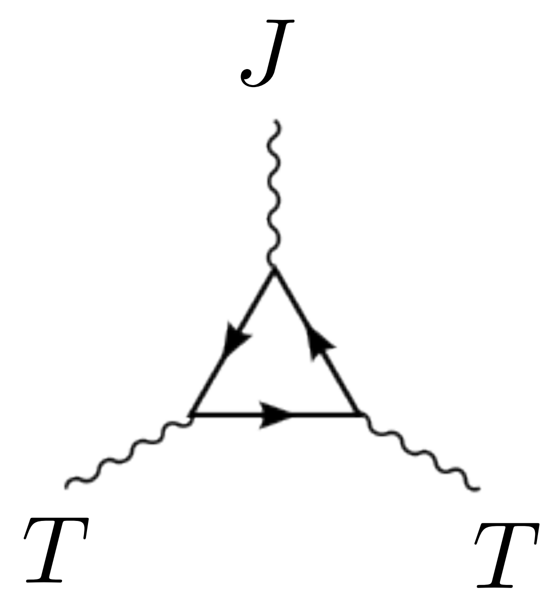

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$$\partial_\mu J^\mu = \frac{1}{4}\epsilon^{\kappa\sigma\alpha\beta}c_M R^\nu{}_{\lambda\kappa\sigma}R^\lambda{}_{\nu\alpha\beta}$$

$$c_M = -\frac{2\pi}{4!(16\pi^3)}\sum_{i=species}\chi_i(q_i)$$

Anomalies

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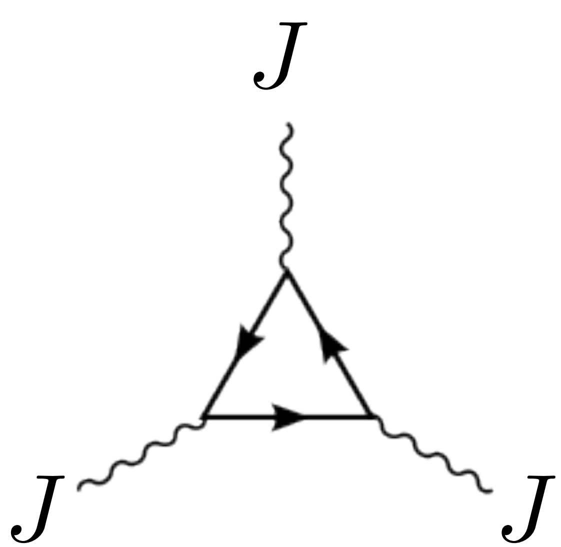

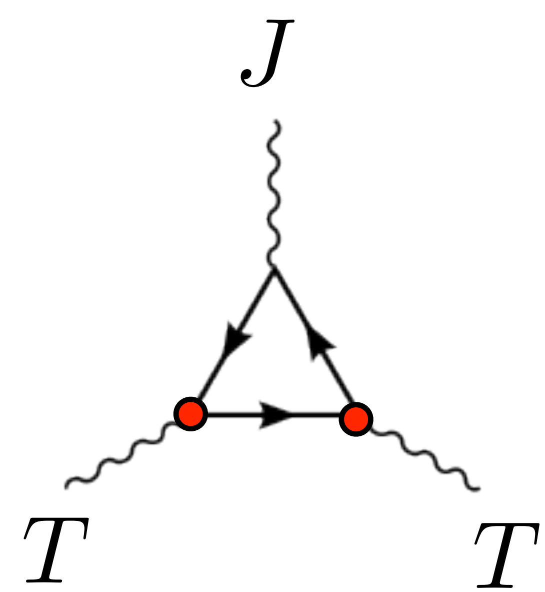

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Anomalies

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \not{D} \psi + \ldots$$

$$\partial_\mu J^\mu = \frac{1}{4} \epsilon^{\kappa\sigma\alpha\beta} \left(3 c_A F_{\mu\nu} F_{\rho\sigma} + c_M R^\nu{}_{\lambda\kappa\sigma} R^\lambda{}_{\nu\alpha\beta} \right)$$

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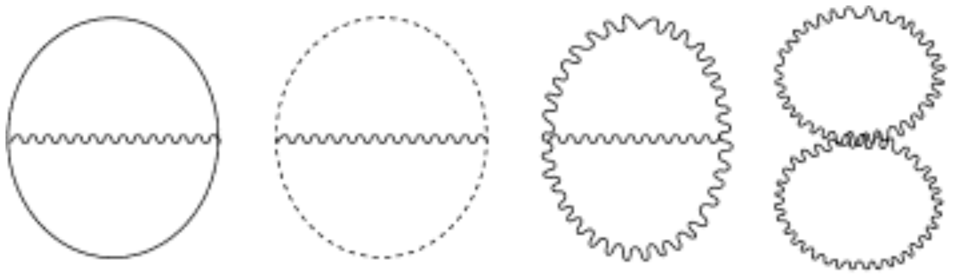

Anomalies and hydrodynamics

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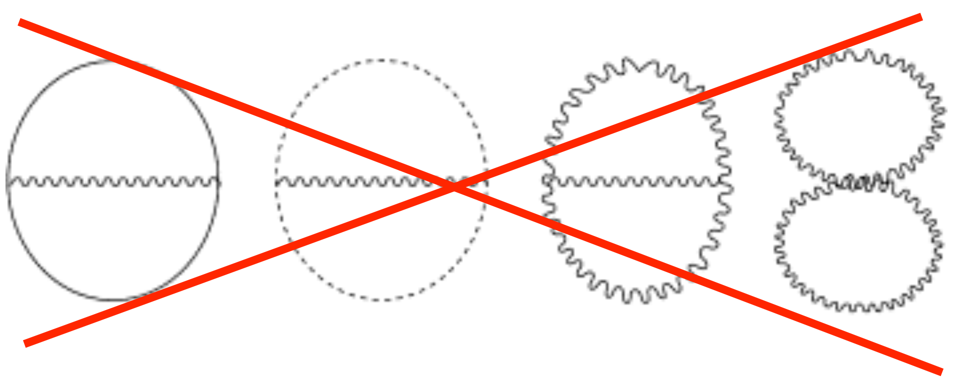
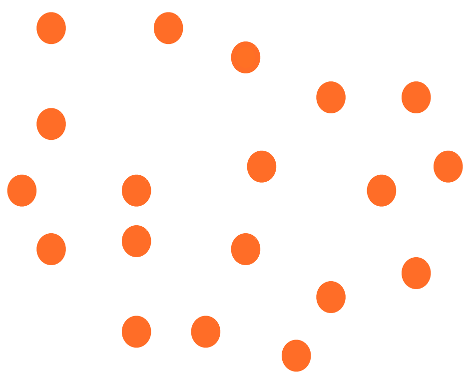


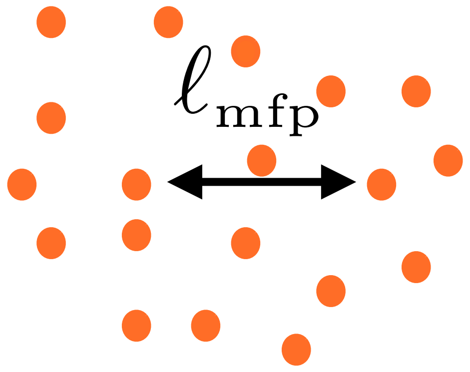
Anomalies and hydrodynamics

Hydrodynamics

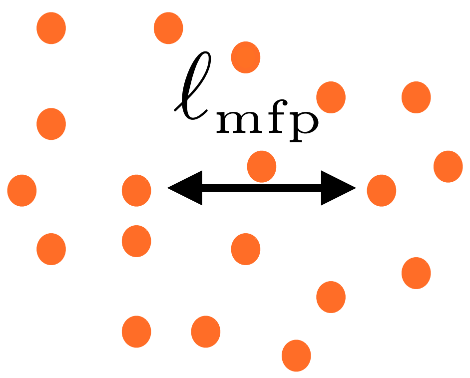
Hydrodynamics



Hydrodynamics

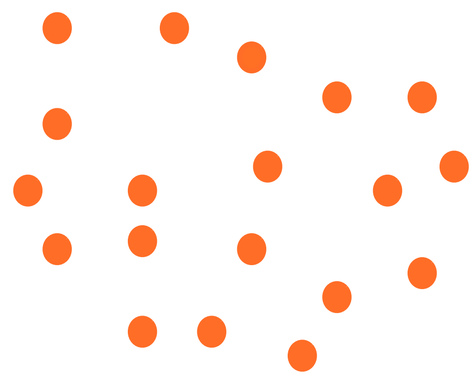


Hydrodynamics



$$L \gg \ell_{\rm mfp}$$

Hydrodynamics



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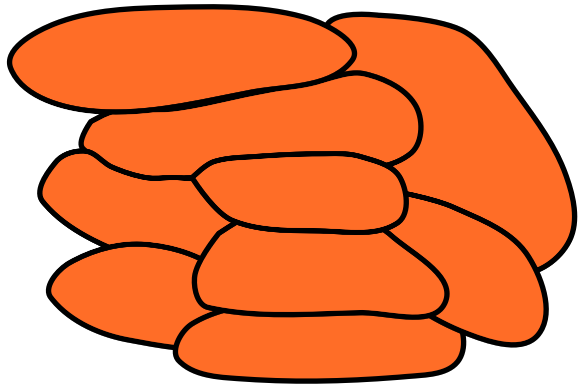

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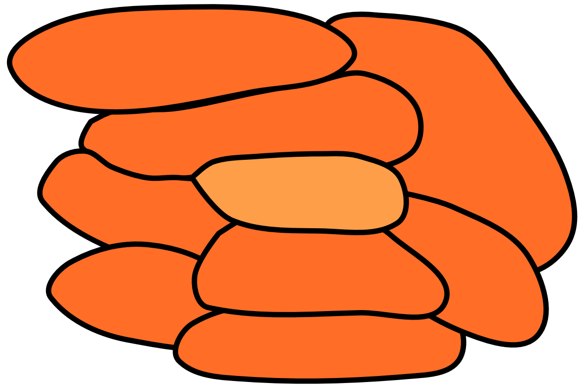

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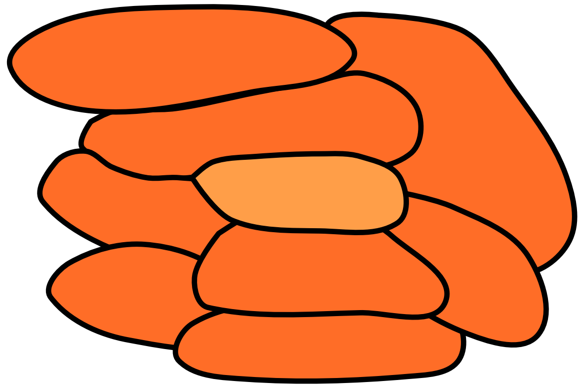
Hydrodynamics

$T(x^{\alpha})$ Temperature



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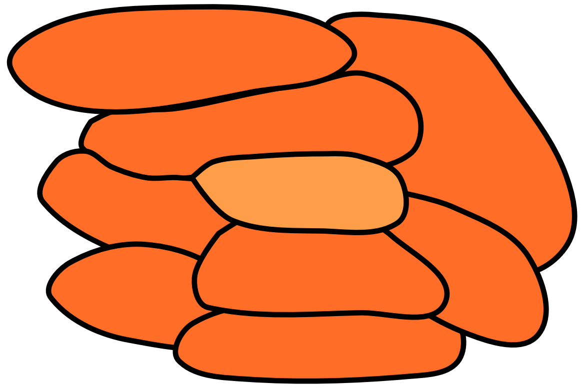
Hydrodynamics

$T(x^\alpha)$ Temperature

$\mu(x^\alpha)$ Chemical potential



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Hydrodynamics

$T(x^\alpha)$ Temperature

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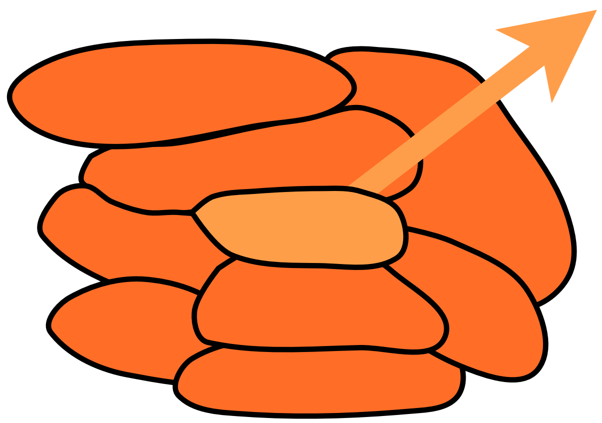
$u^\nu(x^\mu)$ Velocity field



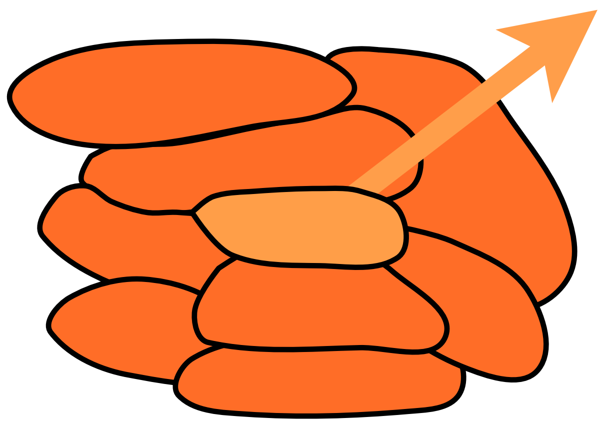
$L \gg \ell_{\mathrm{mfp}}$

Hydrodynamics

$T(x^\alpha)$ Temperature

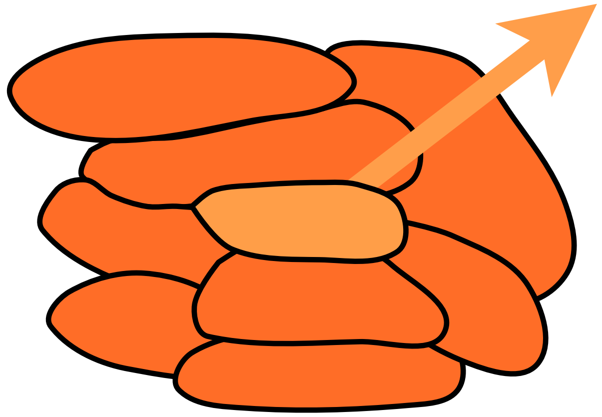
$\mu(x^\alpha)$ Chemical potential

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



$L \gg \ell_{\rm mfp}$

Hydrodynamics



$T(x^\alpha)$ Temperature

$\mu(x^\alpha)$ Chemical potential

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$

$L \gg \ell_{\rm mfp}$

$T^{\mu\nu}[u^\alpha, \mu, T]$

$J^\mu[u^\alpha, \mu, T]$

Hydrodynamics

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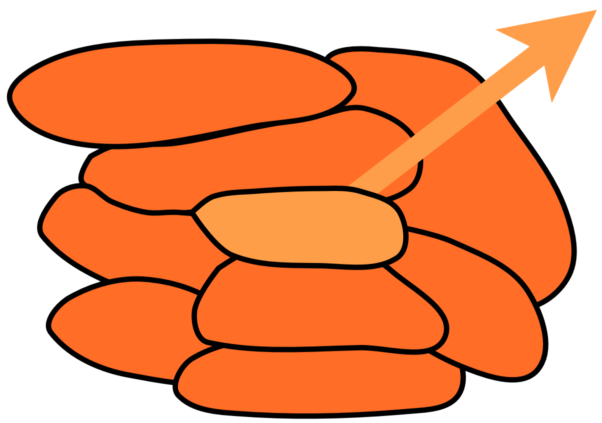

$L \gg \ell_{\rm mfp}$

$T^{\mu\nu}[u^\alpha, \mu, T]$

$J^\mu[u^\alpha, \mu, T]$

$\partial_\mu T^{\mu\nu} = 0$

$\partial_\mu J^\mu = 0$

Hydrodynamics

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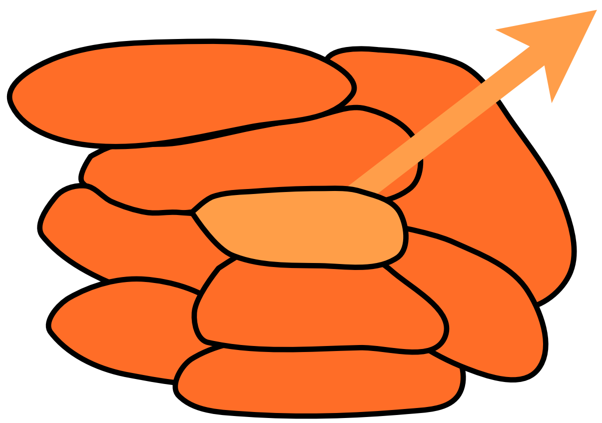
$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



$L \gg \ell_{\mathrm{mfp}}$

To leading order the fields are uniform.

$$T^{\mu\nu}[u^\alpha, \mu, T]$$

$$J^\mu[u^\alpha, \mu, T]$$

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Hydrodynamics

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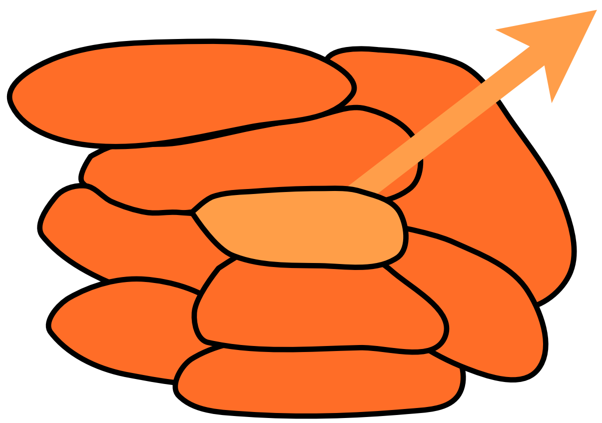

$L \gg \ell_{\mathrm{mfp}}$

To leading order the fields are uniform.

$$T^{\mu\nu}[u^\alpha, \mu, T]$$

$$J^\mu[u^\alpha, \mu, T] = \rho(T, \mu) u^\mu$$

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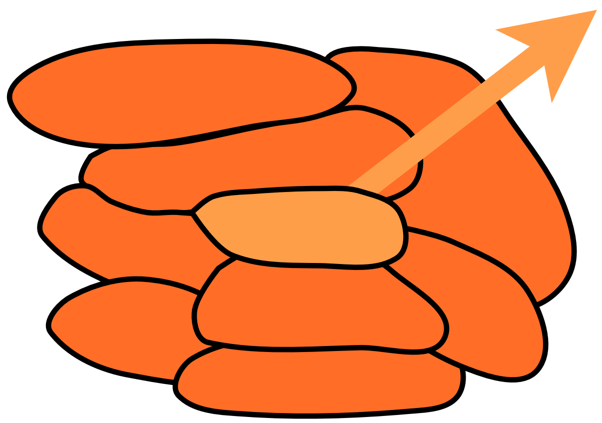
$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



$$L \gg \ell_{\rm mfp}$$

To leading order the fields are uniform.

$$T^{\mu\nu}[u^\alpha, \mu, T]$$

$$J^\mu[u^\alpha, \mu, T] = \rho(T, \mu)u^\mu + \mathcal{O}(\partial)$$

$$\partial_\mu T^{\mu\nu} = 0$$

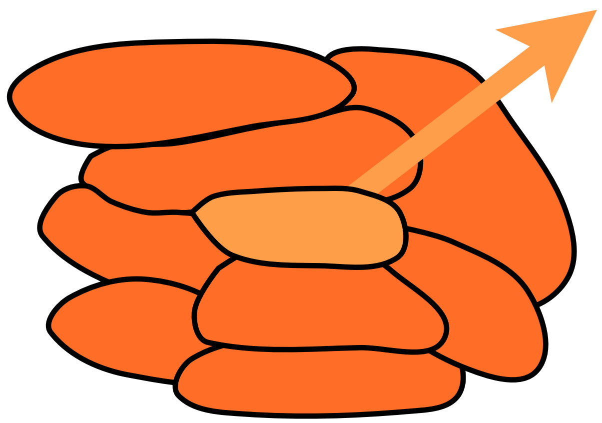
$$\partial_\mu J^\mu = 0$$

Hydrodynamics

$T(x^\alpha)$ Temperature

$\mu(x^\alpha)$ Chemical potential

$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



$L \gg \ell_{\rm mfp}$

To leading order the fields are uniform.

$$T^{\mu\nu}[u^\alpha, \mu, T] = \epsilon(T,\mu)u^\mu u^\nu + P(T,\mu)(u^\mu u^\nu + \eta^{\mu\nu}) + \mathcal{O}(\partial)$$

$$J^\mu[u^\alpha, \mu, T] = \rho(T,\mu)u^\mu + \mathcal{O}(\partial)$$

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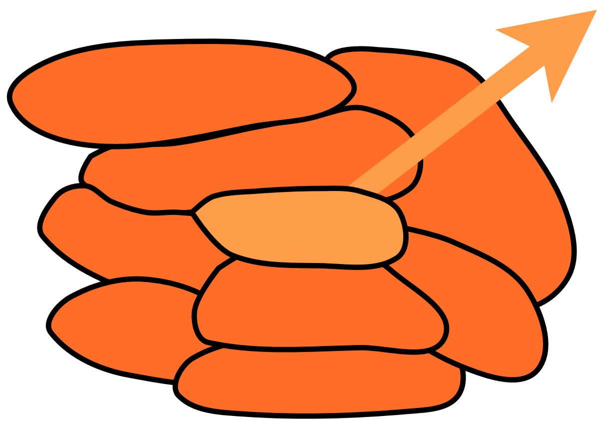

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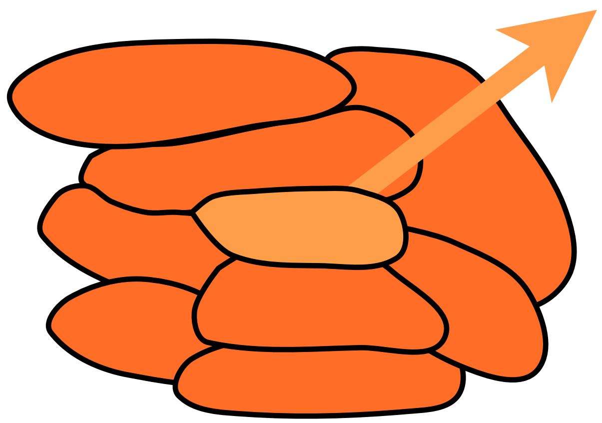

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$$J^\mu[u^\alpha, \mu, T] = \rho(T,\mu)u^\mu + \mathcal{O}(\partial)$$

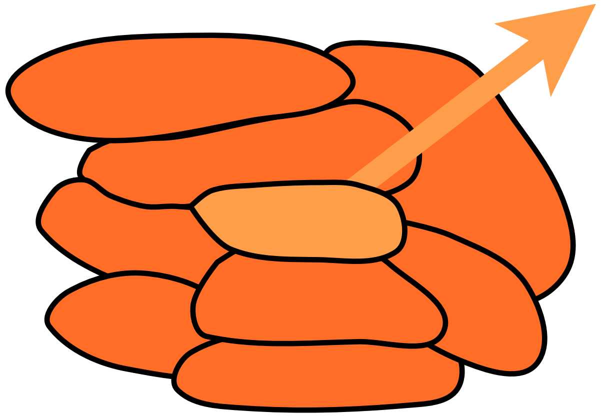
$$u^\mu u^\nu + \eta^{\mu\nu} = P^{\mu\nu}$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

Hydrodynamics

$T(x^\alpha)$	Temperature
$\mu(x^\alpha)$	Chemical potential
$u^\nu(x^\mu)$	Velocity field $\quad (u_\mu u^\mu = -1)$



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$$J^\mu[u^\alpha, \mu, T] = \rho(T,\mu)u^\mu + \mathcal{O}(\partial)$$

$$\partial_\mu T^{\mu\nu} = 0$$

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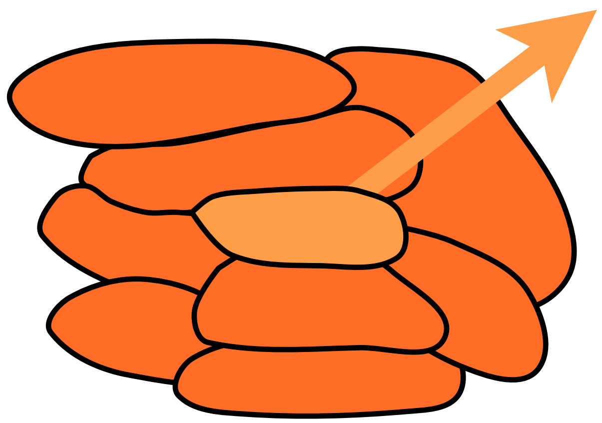
Hydrodynamics



$T(x^\alpha)$	Temperature
$\mu(x^\alpha)$	Chemical potential
$u^\nu(x^\mu)$	Velocity field $(u_\mu u^\mu = -1)$

$$L \gg \ell_{\mathrm{mfp}}$$

At subleading order we allow slowly varying fields

$$T^{\mu\nu}[u^\alpha, \mu, T] = \epsilon(T,\mu) u^\mu u^\nu + P(T,\mu) P^{\mu\nu} + \mathcal{O}(\partial)$$

$$J^\mu[u^\alpha, \mu, T] = \rho(T,\mu) u^\mu + \mathcal{O}(\partial)$$

$$\partial_\mu T^{\mu\nu} = 0$$

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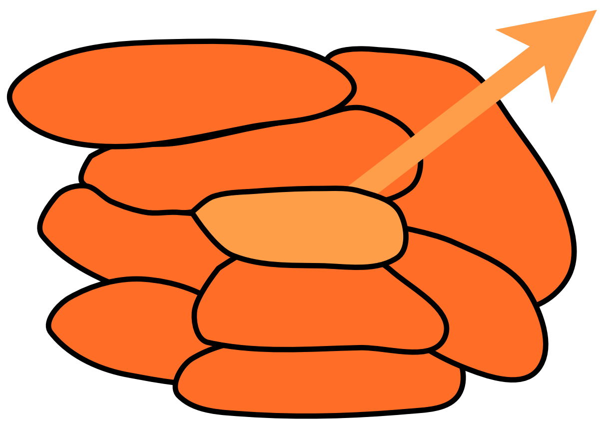

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At subleading order we allow slowly varying fields

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$$J^\mu[u^\alpha, \mu, T] = \rho(T,\mu) u^\mu - \kappa(T,\mu) P^{\mu\nu} \partial_\nu \frac{\mu}{T} + \chi P^{\mu\nu} \partial_\nu T + \theta \omega^\mu$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J^\mu = 0$$

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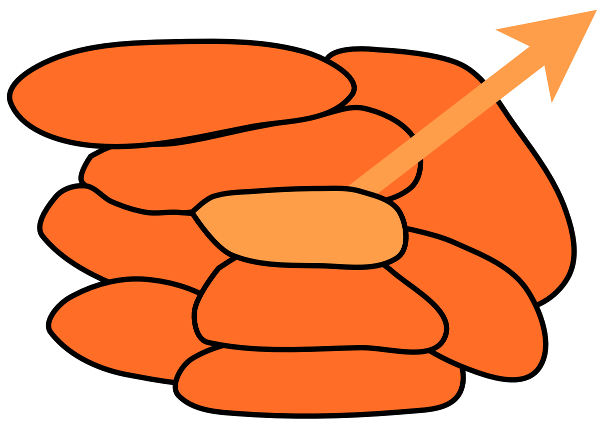
$u^\nu(x^\mu)$ Velocity field $(u_\mu u^\mu = -1)$



$L \gg \ell_{\rm mfp}$

At subleading order we allow slowly varying fields

$$T^{\mu\nu}[u^\alpha, \mu, T] = \epsilon(T,\mu)u^\mu u^\nu + P(T,\mu)P^{\mu\nu} + \mathcal{O}(\partial)$$

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$$\partial_\mu T^{\mu\nu} = 0$$

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$$\omega_{\mu\nu} = \frac{1}{2}P_\mu{}^\alpha P_\nu{}^\beta\left(\partial_\alpha u_\beta - \partial_\beta u_\alpha\right)$$

Hydrodynamics

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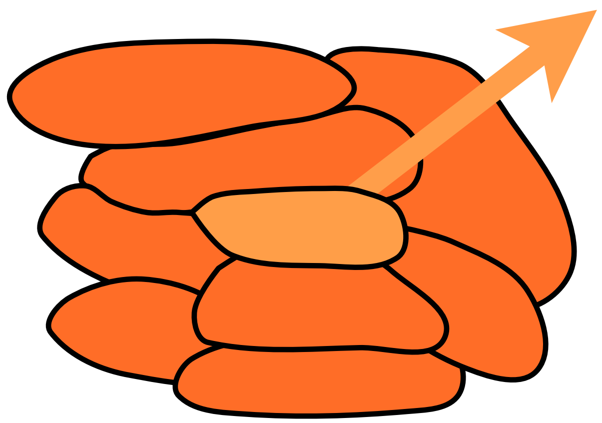
$u^\nu(x^\mu)$	Velocity field $(u_\mu u^\mu = -1)$



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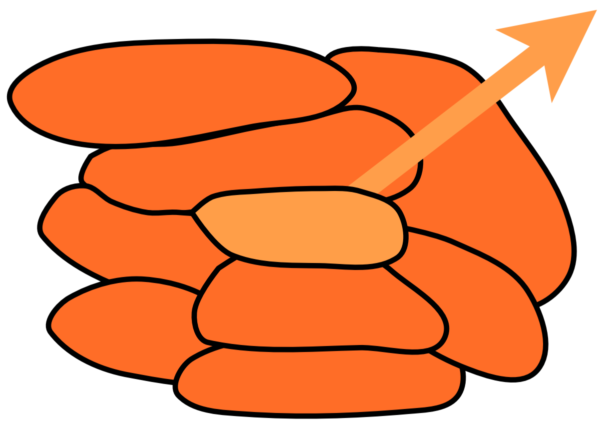
Hydrodynamics



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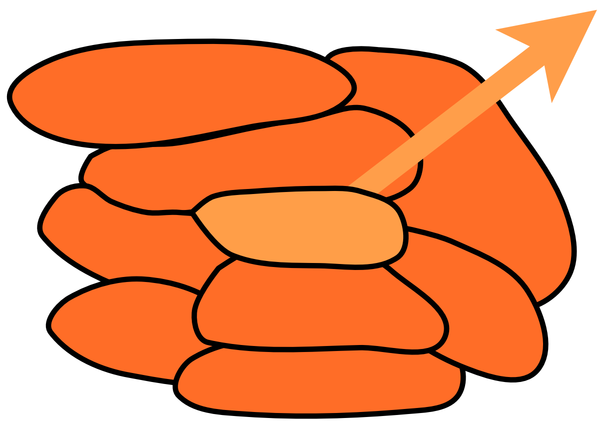

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Thermal conductivity

Hydrodynamics

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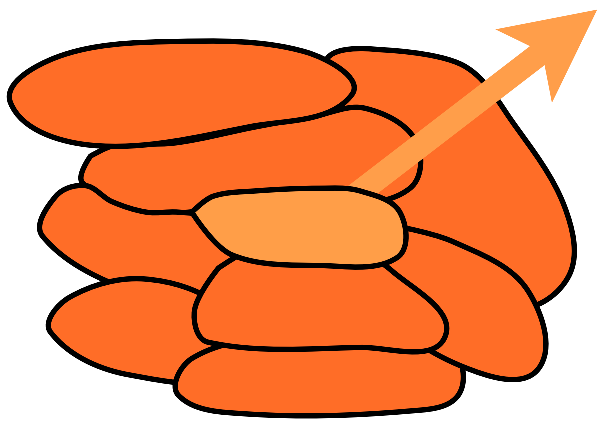

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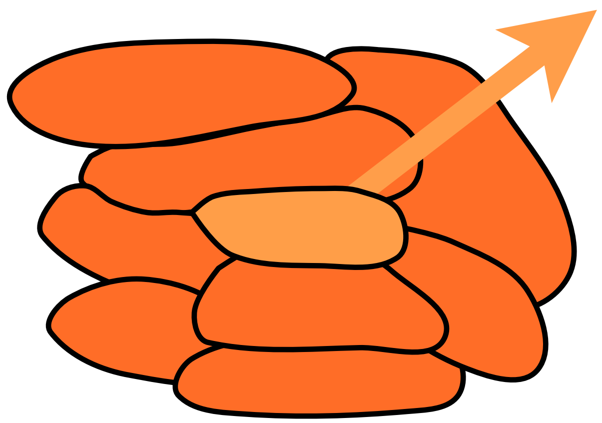

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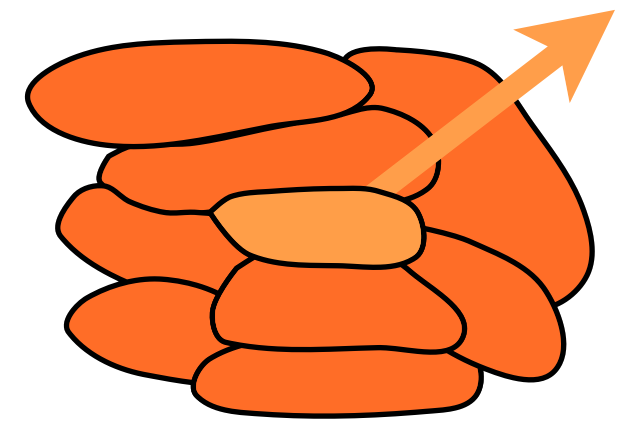
Hydrodynamics



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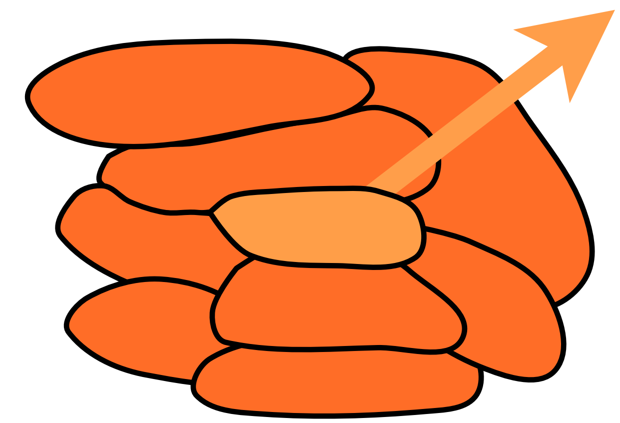

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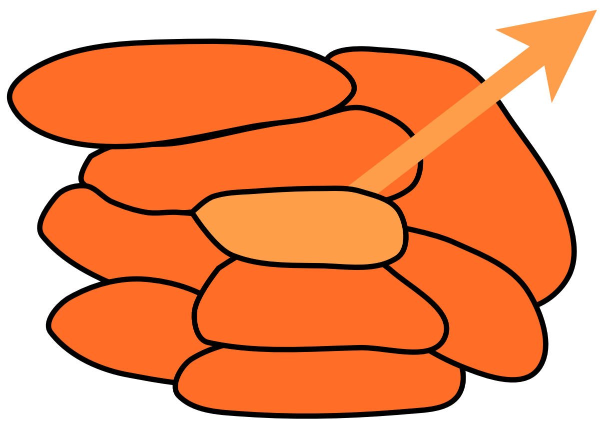
2. Existence of equilibrium configuration
   Jensen, Kaminski, Kovtun, Myer, Ritz, AY (2012)
   Banerjee, Bhattacharya, Bhattacharyya, Jain, Minwalla, Sharma (2012)

Hydrodynamics



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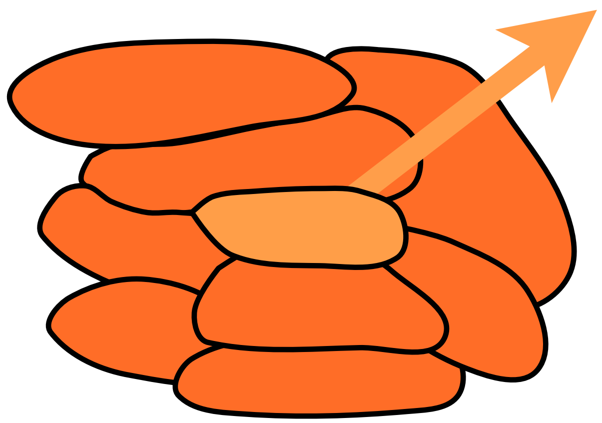

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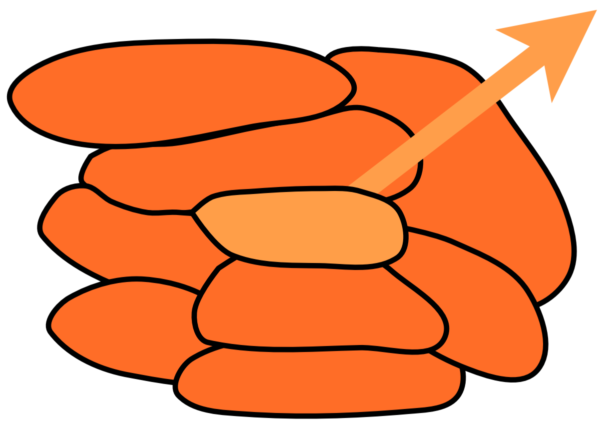

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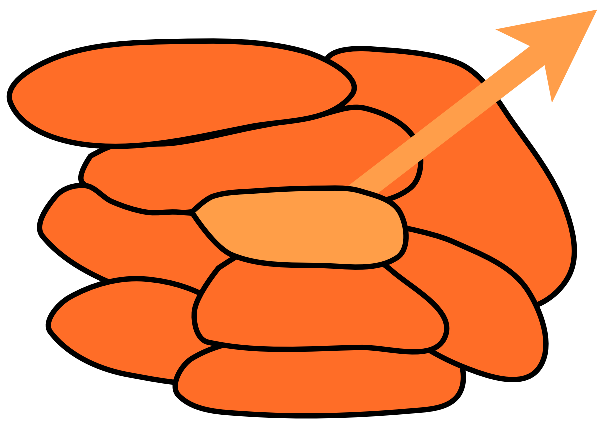

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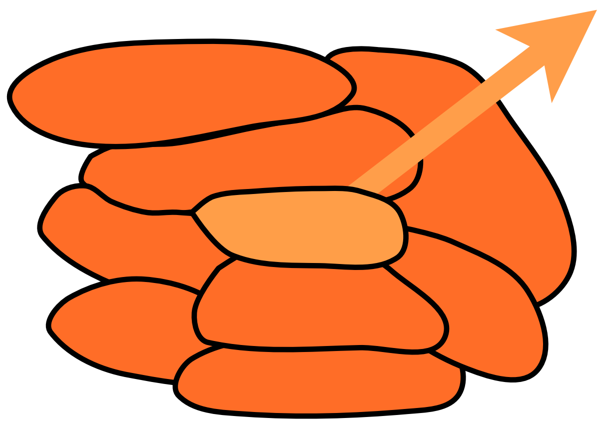

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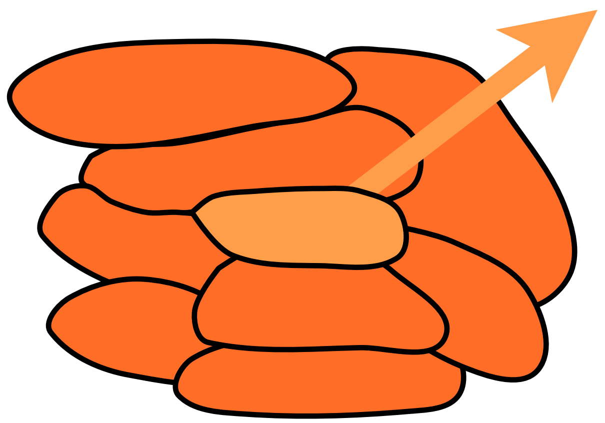

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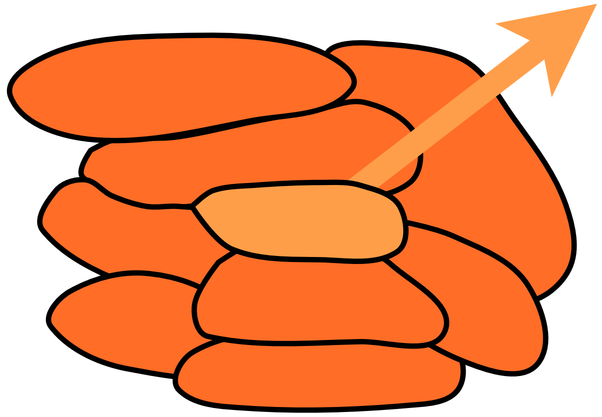

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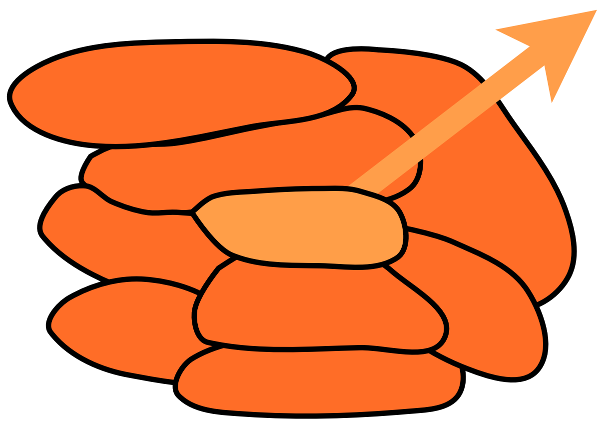

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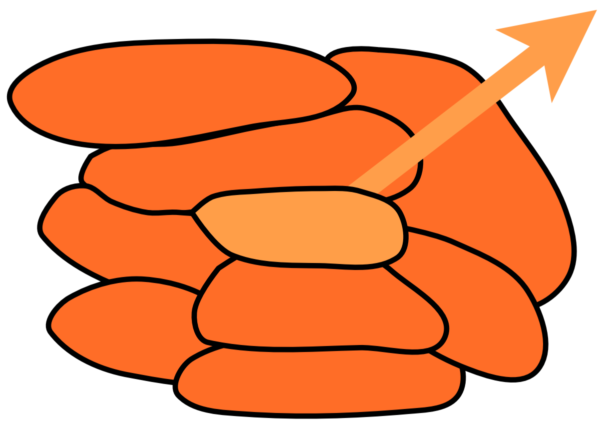

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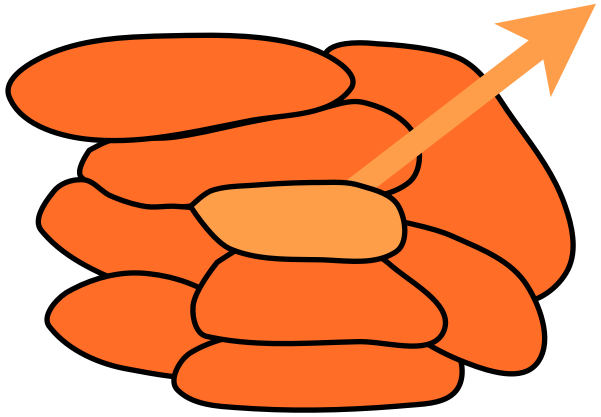

Anomalies and hydrodynamics

$$\partial_\mu J^\mu = \frac{1}{4}\epsilon^{\kappa\sigma\alpha\beta}\left(3c_A F_{\mu\nu}F_{\rho\sigma} + c_M R^\nu{}_{\lambda\kappa\sigma}R^\lambda{}_{\nu\alpha\beta}\right)$$

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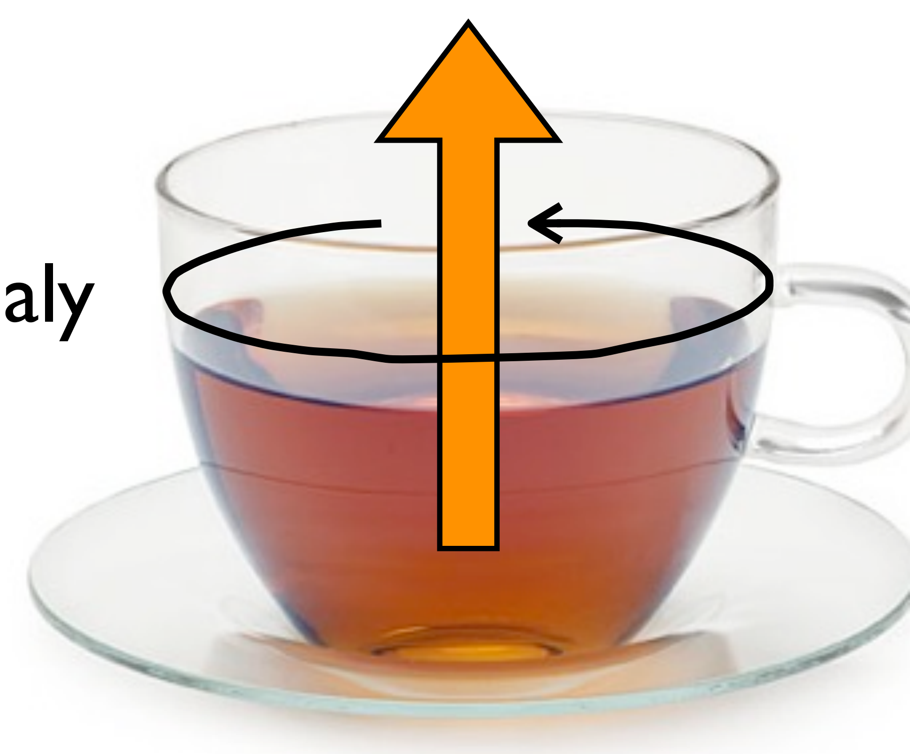
2. Experimental signature of mixed anomaly



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Is there an underlying structure?

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Other terms in 3+1d

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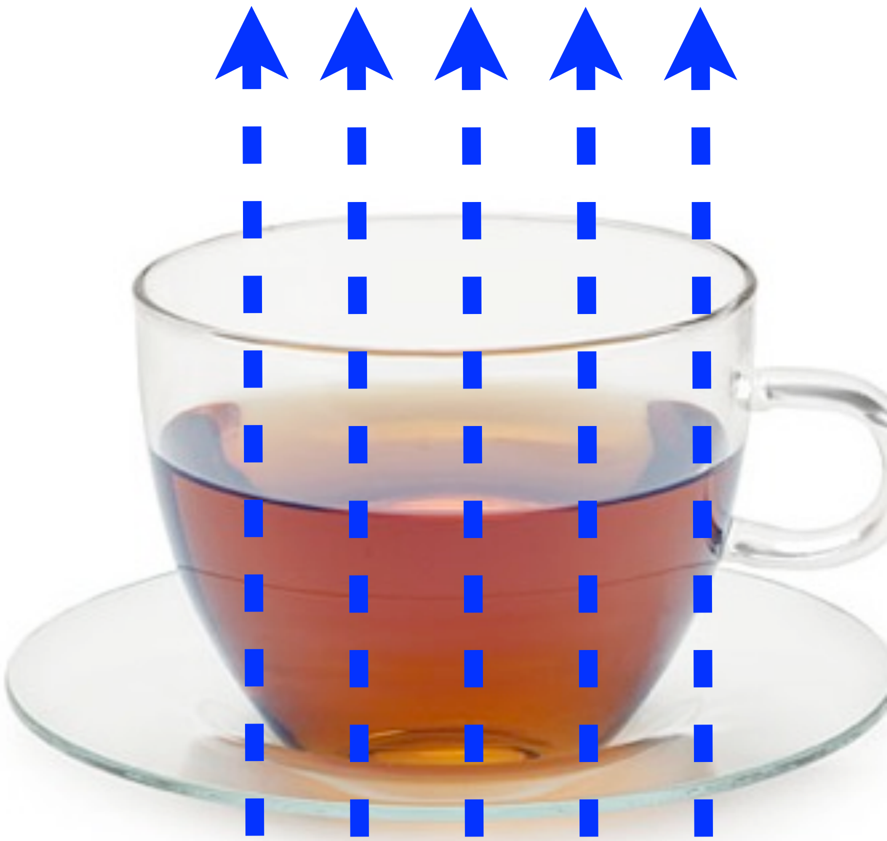

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$$J^\mu \sim \theta \omega^\mu$$

$$\theta = -8\pi^2 c_m T^2 - 3 c_A \mu^2$$

$$B_{\mu\nu} = P_{\mu\alpha} P_{\nu\beta} F^{\alpha\beta}$$

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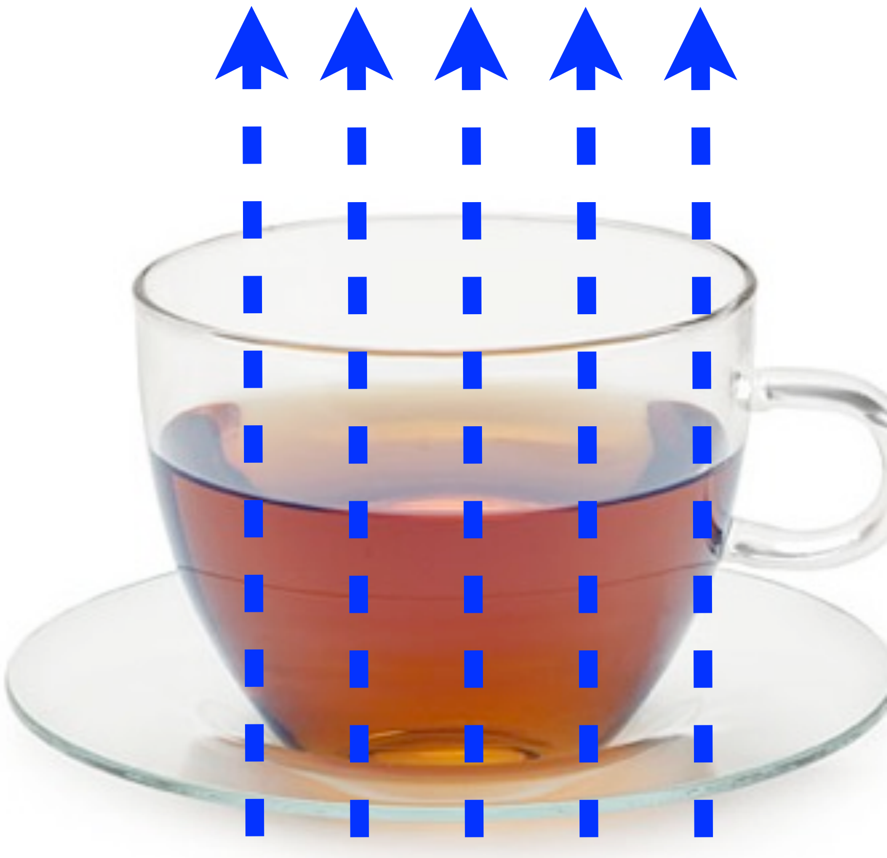

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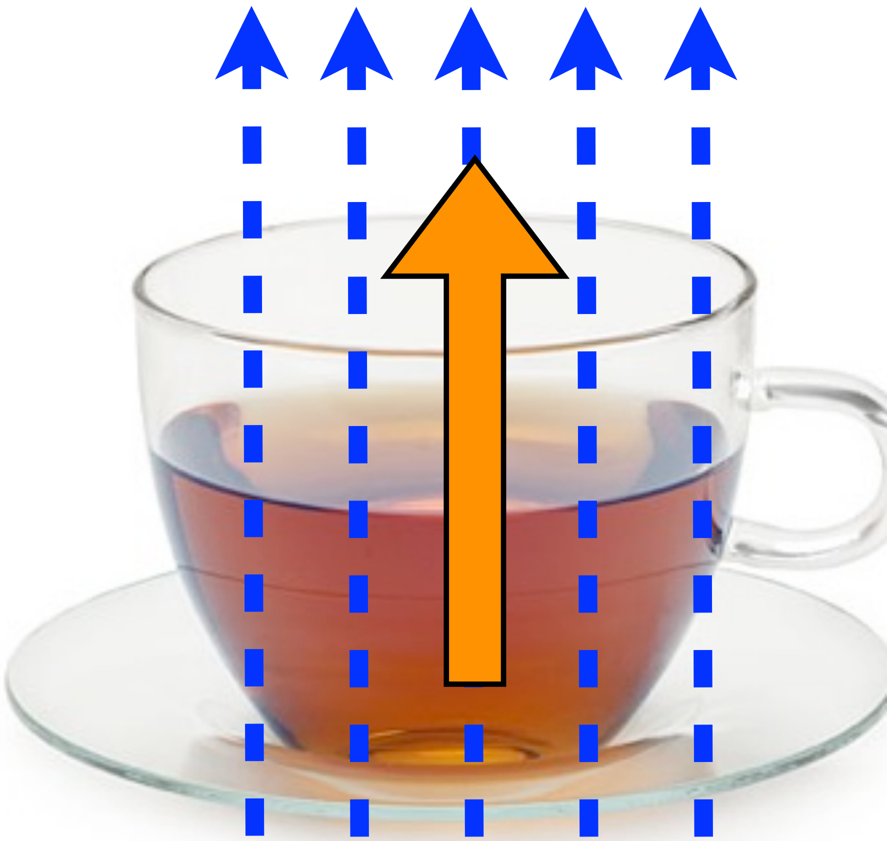

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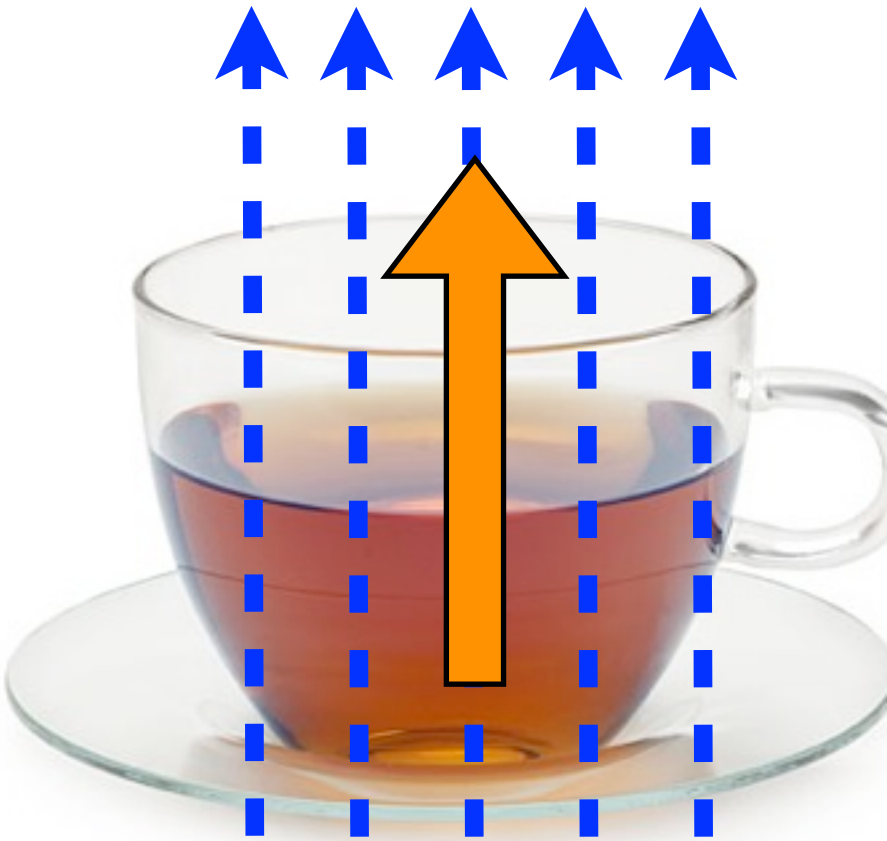

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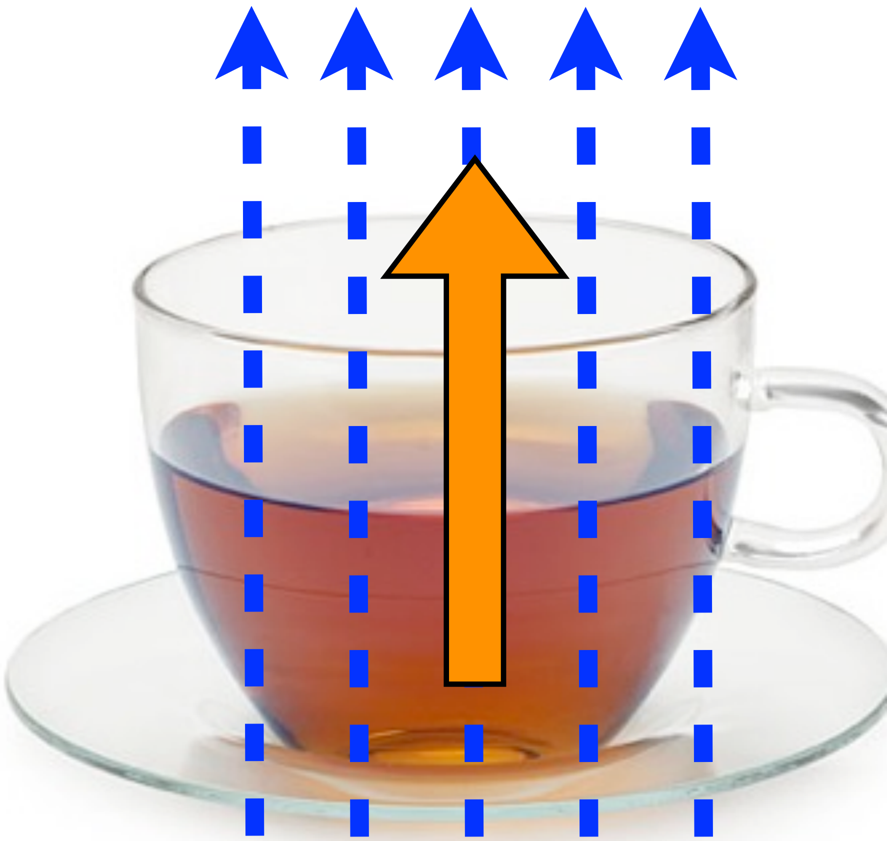

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Other terms in 3+1d

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A master function

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$$\mathbf{P}\left(\mathrm{Tr}(\mathbf{R}^{2n}), \mathbf{F}^{m}\right)$$

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$$\mathbf{A}_T \qquad \mathbf{F}_T = d\mathbf{A}_T \qquad \mu_T = 2\pi T$$

Construct a thermal anomaly polynomial:

$$\mathbf{P}_T = \mathbf{P}\left(\mathrm{Tr}(\mathbf{R}^{2n}) + 2\mathbf{F}_T^{2n},\, \mathbf{F}^m\right)$$

Introduce hatted field strengths:

$$\hat{\boldsymbol{F}} = \boldsymbol{B} + 2\boldsymbol{\omega}\mu \qquad \hat{\mathbf{F}}_T = \mathbf{B}_T + 2\boldsymbol{\omega}\mu_T$$

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Anomaly polynomial in 3+1 dimensions

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Anomaly polynomial in 1+1 dimensions

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For a formal proof see:

Jensen, Loganayagam, Yarom 1308.xxxx

Jensen, Loganayagam, Yarom 1309.xxxx

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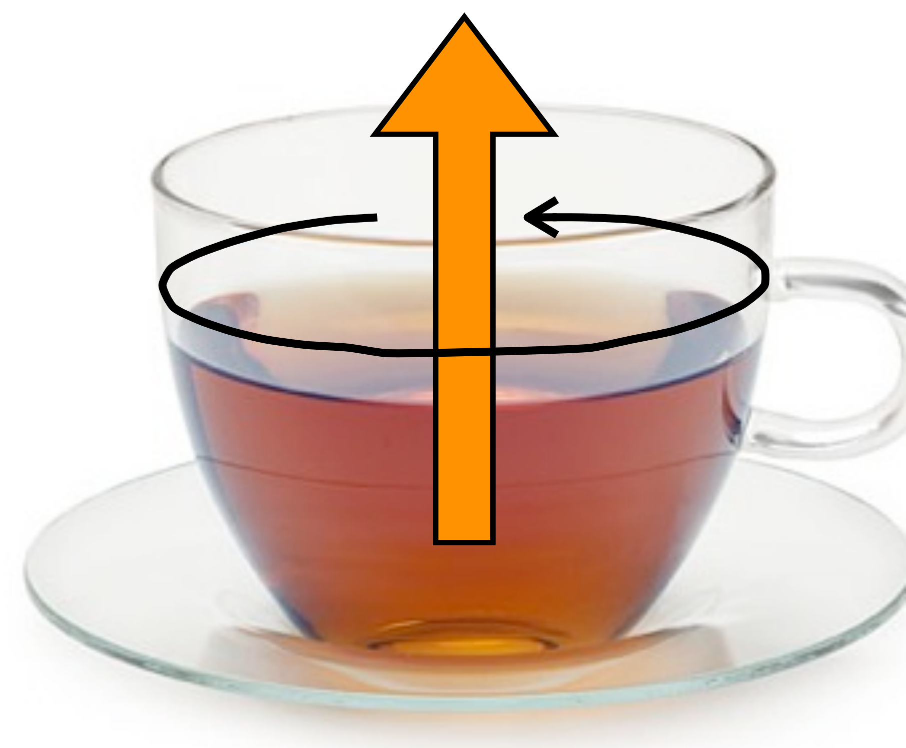

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Thank you