The sector with $g>0$ acquires a gap.
Some operators acquire vacuum averages
This enhances response ~~of~~ to certain perturbations.

How to characterize WZNW theory?

1. This theory is critical. (1+1) one.
2. Critical (1+1) - theories ~~have~~ possess conformal symmetry.

This means that on infinite plane their correlation functions ~~are power law~~ decay as power law.

It also means that holomorphic and antiholomorphic sectors ($z = \tau + ix$, $\bar{z} = \tau - ix$) are separated

$$\langle\langle O_1(1) \dots O_N(N) \rangle\rangle = \sum C_{ij} \mathcal{F}_i(z_1, \dots z_N) \bar{\mathcal{F}}_j(\bar{z}_1, \dots \bar{z}_N)$$

$$O_\Delta(\xi) = O_\Delta[z(\xi)] \left(\frac{dz}{d\xi}\right)^\Delta$$

There are operator = primary fields transforming under holomorphic transformations as

Eigen States $\leftrightarrow$ operators

C2

States $\leftrightarrow$ operators.

$$\langle 0|\mathcal{O}(\tau,x)\,\mathcal{O}^{+}(0,0)|0\rangle = \sum_n \langle 0|\mathcal{O}(x,\tau)|n\rangle\langle n|\mathcal{O}^{+}(0,0)|0\rangle$$

$$\mathcal{O}(x,\tau)|n\rangle = e^{-\tau E_n - iP_n x}\,\mathcal{O}(0,0)|n\rangle$$

Lehmann expansion.

On the other hand, in ~~critical~~ conformal theories on stripe $x \in (0,L)$

$$\langle\langle \mathcal{O}(\tau,x)\,\mathcal{O}^{+}(0,0)\rangle\rangle = \left\{\frac{\pi}{L\sinh\frac{\pi}{L}(\tau v + ix)}\right\}^{2\Delta} \times \left\{\frac{\pi}{L\sinh\frac{\pi}{L}(\tau v - ix)}\right\}^{2\bar{\Delta}}$$

Let $\tau > 0$, expand

$$\left(\frac{\pi}{L}\right)^{2(\Delta+\bar{\Delta})} e^{-\frac{2\pi}{L}(\Delta+\bar{\Delta})\tau v - i\frac{2\pi(\Delta-\bar{\Delta})}{L}x} \times$$

$$\times \sum_{N>0} C_{nm}\, e^{-\frac{2\pi}{L}N\tau v - i\frac{2\pi}{L}Mx}$$

$$E(J,j;n) = \frac{2\pi v}{L}\left(\Delta_J + \bar{\Delta}_J + N\right)$$

$$\Delta_J = \frac{1}{k+2}J(J+1)$$

$$J^{a_1}_{-q_1}\dots J^{a_N}_{-q_N}|J,j\rangle$$

$$N = q_1 + \dots q_N$$

$$J(x) = \frac{1}{L}\sum_q J_q\, e^{-\frac{2\pi i q}{L}}$$

~~$q = -\infty$~~

q - integer.

Is ~~mod~~ WZNW model tractable?

Yes, the Sugawara Hamiltonian can be diagonalized

Introduce vacuum states which satisfy:

$$J^a_{+n} |v\rangle = 0, \quad n > 0$$

$$\vec{J}_0^{\,2} |v\rangle = \mathbb{C} |v\rangle$$

↑ quadratic Casimir.

For simplicity, I'll consider $G = SU(2)$

$$|v\rangle = |J, j\rangle$$

$$J_0^3 |J, j\rangle = j |J, j\rangle$$

$$\vec{J}^{\,2} |J, j\rangle = j(j+1) |J, j\rangle$$

$J^{a_1}_{-n_1} J^{a_2}_{-n_2} \dots J^{a_N}_{-n_N} |J, j\rangle$ are eigenstates

$$E_n = \frac{2\pi}{(k+2)L}\left[j(j+1)\right] + \text{integer} \; \frac{2\pi}{L}$$

In the thermodynamic limit we have a linear spectrum $\varepsilon \sim |q|$

Specific heat $$\frac{C_v}{T} = \frac{\pi^2}{3} \cdot \frac{3k}{k+2}$$

Central charge

Lagrangian form.

$$W[g] = \frac{1}{16\pi}\int d^2x \, Tr\left(\partial_\mu g^{-1}\partial_\mu g\right) + \Gamma[g].$$

$$\Gamma[g] = -\frac{i}{24\pi}\int_0^\infty d\xi \underbrace{\int d^2x\, \epsilon^{\alpha\beta\gamma}\, Tr\left[g^{-1}\partial_\alpha g\, g^{-1}\partial_\beta g\, g^{-1}\partial_\gamma g\right]}_{\text{Total derivative in } \xi}$$

$$g(\xi=0, x) = g(x)$$
$$g(\infty, x) = I.$$

$SU(2)$

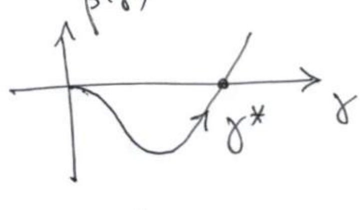
$$g = \underline{n_0 \hat{I}} + i\vec{\sigma}\cdot\vec{n}, \qquad n_0^2 + \vec{n}^2 = 1.$$

$S = kW[g]$ — critical WZW.

$$\mathcal{L}_{general} = \frac{1}{16\pi\gamma} Tr\left(\partial_\mu g^{-1}\partial_\mu g\right) + k\Gamma[g]$$

General WZNW

$$\dot{\gamma} = N\gamma^2 - kN\gamma^3 + \dots$$



Close to criticality

$$kW[g] + \lambda\int :J^a\bar{J}^b\Phi_{ab}: d^2x$$

$$d = \frac{2C_v}{k+2} + 2$$

<u>irrelevant</u> operator.

Application of magnetic field:

$$J_0^z = \frac{1}{L}\int J^3(x)\,dx \sim \chi H$$ becomes finite.

$$J^a \bar{J}^b \Phi_{ab} \to \chi^2 H^2 \Phi_{zz}$$

relevant operator !

$$KW[g] + \lambda \Phi_{zz}$$

Integrable theory !

Conformal embedding

$$SU_k(2) = U(1) \otimes Z_k$$

critical model of parafermions.

$$J^3 = \sqrt{\frac{k}{2\pi}}\,\partial_z \varphi$$

$$J^\pm = \frac{1}{2\pi a_0} e^{\pm i \sqrt{\frac{8\pi}{k}}\varphi}\, \psi^\pm$$

by definition this is a parafermion operator.

$$\langle \psi(z)\psi(0)\rangle = \frac{1}{z^{2(1-1/k)}}$$

It is nonlocal except for $k=2$ whe it is a Majorana fermion.

$$SU_k(N) = [U(1)]^{N-1} \otimes \frac{SU_k(N)}{[U(1)]^{N-1}}$$

Gepner's parafermions.

Z_k - generalization of the Ising model

$\sigma, \sigma^2, \dots \sigma^{k-1}$ $\sigma^{k-n} = (\sigma^n)^+$

order parameters.

Adjoint rep $\phi^{ab} = \mathrm{Tr}(\sigma^a g \sigma^b g^+)$

Semiclassical approximation

$$\mathrm{Tr}\,\hat{\phi} = n_0^2$$

Depending on the sign the coupling λ

$$k W[g] + \lambda \mathrm{Tr}\,\phi^{adj}$$

we get either $n_0 = 0$, or $\vec{n}^2 = 0$.

as the ground state.

$$\frac{k}{8\pi}(\partial_\mu \vec{n})^2 + |\lambda| \vec{n}^2 + \dots \qquad \lambda < 0$$

$$n^2 << 1.$$

Massive triplets.

$$\frac{k}{8\pi}(\partial_\mu \vec{n})^2 + i\pi k\,\Theta$$

topological term.

Various representations of the Kac-Moody algebras.

$SU_1(2)$ can be obtained by the ordinary Abelian bosonization.

$$R_\sigma = \frac{1}{\sqrt{2\pi a_0}} e^{i\sqrt{4\pi}\varphi_\sigma} \chi_\sigma \qquad \{\chi_\sigma, \chi_{\sigma'}\} = \delta_{\sigma\sigma'},$$

$$L_\sigma = \frac{1}{\sqrt{2\pi a_0}} e^{-i\sqrt{4\pi}\bar{\varphi}_\sigma} \chi_\sigma$$

$$\varphi_\sigma = \frac{1}{\sqrt{2}}(\varphi_c + \sigma\varphi_s), \qquad \sigma = \pm 1.$$

$$j^3 = \frac{1}{2}\left(R_\uparrow^+ R_\uparrow - R_\downarrow R_\downarrow\right)$$

$$j^\pm = R_\uparrow^+ R_\downarrow, \quad R_\downarrow^+ R_\uparrow$$

$$\left\{ \begin{aligned} J^+ &= \frac{i}{2\pi a_0} e^{-i\sqrt{8\pi}\varphi} \\ J^- &= \frac{-i}{2\pi a_0} e^{i\sqrt{8\pi}\varphi} \\ J^z &= \frac{i}{\sqrt{2\pi}} \partial_z \varphi \end{aligned} \right.$$

$$z = \tau + ix.$$

$$S = \int d\tau dx\, \partial_x\varphi\left(-i\partial_\tau\varphi + \partial_x\varphi\right)$$