AN INTRODUCTION TO THE SPECIES SCALE IN QUANTUM GRAVITY & STRING THEORY

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REFERENCES (incomplete)

hep-th/0106058 DVALI, GABADADZE, KOLANOVIC, NITTI
hep-th/0110129 VENEZIANO
hep-ph/0404182 HAN, WILLENBROCK
0710.4344 DVALI, REDI
0912.3167 DVALI, LÜST
1004.3744 DVALI, GOMEZ
2212.03908 CASTELLANO, HERRAEZ, IBANEZ
2212.06841 VAN DE HEISTEEG, VAFA, WIESNER, WU
2212.10286 CRIBIORI, LÜST, STAUDT
2303.13580 VAN DE HEISTEEG, VAFA, WIESNER, WU
2305.10489 CRIBIORI, LÜST, MONTELLA
2305.10490 BLUMENHAGEN, GLIGOVIC, PARASKEVOPOULOU
2306.08673 CRIBIORI, LÜST

These notes have been prepared for personal use. As such, they may contain typos, mistakes, misconceptions, ... Whenever something is unclear, the reader should look at the appropriate references mentioned in the text.

Refs: hep-th/9401070 eq. 3.14
hep-th/0002145 eq 5.3
hep-th/0106058 eq 1.5
hep-th/0110129 eq 2
hep-ph/0404182 eq 6

• INTRODUCTION AND MOTIVATION

What is the species scale?
Why is it important?

The PLANCK SCALE $M_P = \left(\frac{\hbar c}{G_N}\right)^{\frac{1}{2}} = \frac{1}{L_P} \simeq 10^{19}\, GeV/c^2$

was proposed by Planck as a fundamental scale of Nature, with the UNITS of MASS (or LENGTH if L_P).
Dimensional analysis tells us that we can organize a perturbative expansion in gravitational EFTs in terms of the coupling

$$E/M_P$$

Then, the EFT breaks down when $E \sim M_P$.
However, dimensional analysis by definition is BLIND to DIMENSIONLESS COUPLINGS. In other words, dimensional analysis cannot really tell us if the EFT breaks down, say, at $E \sim M_P \sim 10^{19}$ GeV or $E \sim \frac{M_P}{\sqrt{8\pi}} \sim 10^{18}$ GeV or perhaps at some scale way smaller.

For this reason, at page 1 of hep-ph/0404182 they write "We still do not know the true physical meaning of the Planck mass". Then, they propose to determine the DIMENSIONLESS COUPLING possibly entering $E \sim M_P$ by demanding that the EFT breaks down when it violates UNITARITY.

[Instead, Veneziano in hep-th/0110129 recovers the species scale from (eq. 11)

$$4\pi\alpha_G \equiv \kappa^2 \Lambda^{D-2} \sim \left[\kappa_0^{-2}\Lambda^{2-D} + (C_0 N_0 + C_{1/2} N_{1/2} + C_1 N_1)\right]^{-1} \leq (C_0 N_0 + C_{1/2} N_{1/2} + C_1 N_1)^{-1}$$

RENORMALIZED GRAV. COUPLING

and by taking the limit of LARGE BARE COUPLING, $\kappa_0 \to \infty$, in which the bound is saturated]

Briefly, in hep-th/0404182 they use the fact that the partial-wave amplitude, a_J, for a $2 \to 2$ elastic scattering is UNITARY when $|\mathrm{Re}\, a_J| \leq \frac{1}{2}$. They calculate all $2\to 2$ scattering amplitudes in QUANTUM GRAVITY and ask at what scale the bound is violated. This scale is the species scale.

SPIN

From an explicit computation, upon converting the amplitude A into partial waves

WIGNER FUNCTIONS

$$A = 16\pi \sum_J (2J+1)\, a_J\, d^J_{\mu\mu}$$

SPIN

they find (eq 5)

$$a_2 = -\frac{1}{40} G_N E_{CM}^2 N \quad , \quad N = \frac{2}{3} N_s + N_f + 4 N_V$$

SPIN J=2

Then

$$|\mathrm{Re}\, a_J| = \frac{1}{2} \implies E_{CM}^2 = 20 (G_N N)^{-1} < \frac{1}{G_N} = M_P^2$$

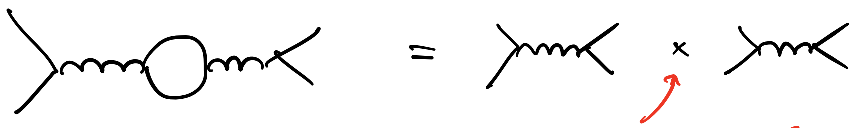
$\hbar = c = 1$

In Standard Model with Higgs doublet and 3 generations of fermions, $N_s = 2$, $N_f = 45$, $N_V = 12$ and unitarity is violated at $E_{CM} = \sqrt{\frac{60}{283}}\, G_N^{-1/2} \simeq 6 \cdot 10^{18}$ GeV $\simeq \frac{1}{2} M_P$

- ORIGIN of COEFFICIENT N: The unitarity condition is actually

$$\text{Im}\, a_J \geq |a_J|^2$$

which implies $|\text{Re}\, a_J| \leq \frac{1}{2}$. The above equation is interpreted diagrammatically as (when saturated)



SUM over the N particles running in the loop, since I have to take into account all the combinations.

A similar PERTURBATIVE ARGUMENT has been proposed in hep-th/0710.4344 and also reviewed recently in 2212.03908. From the latter, we can read off the 1-loop propagator of the graviton coupled to N MASSLESS SCALARS

$$; \quad \Pi^{\mu\nu\rho\sigma} = i\left(P^{\mu\nu}P^{\nu\sigma} + P^{\mu\sigma}P^{\nu\rho} - P^{\mu\nu}P^{\rho\sigma}\right)\pi(p^2)$$

$$\text{where} \qquad P^{\mu\nu} = \eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}$$

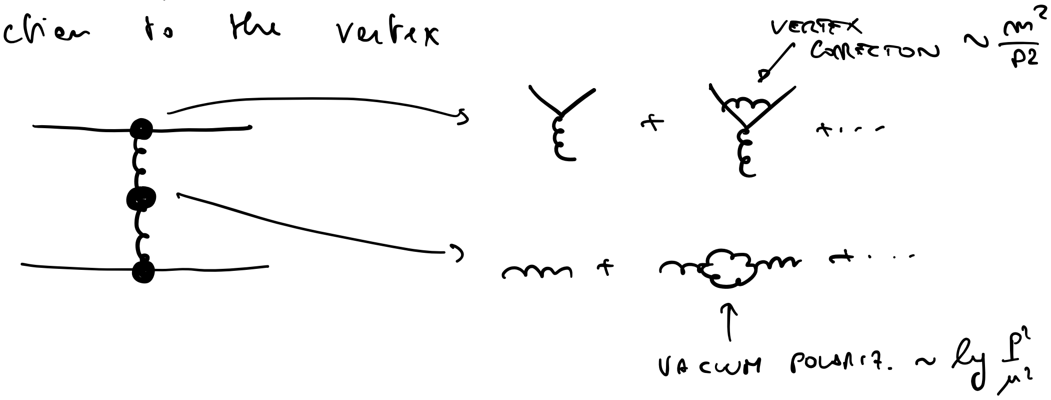
$$\pi^{-1}(p^2) = 2p^2\Big(\underbrace{1}_{\text{TREE LEVEL}} - \underbrace{\frac{Np^2}{120\pi M_p^2}\log\left(-\frac{p^2}{\mu^2}\right)}_{\text{1-LOOP}}\Big)$$

Perturbation theory breaks down when tree level = 1-loop, which happens at a scale

(see eq. 2.5 of 2212.03908)
$$p^2 \sim \Lambda_{sp}^2 \sim \frac{M_p^2}{N} \qquad \text{SPECIES SCALE}$$

up to LOGARITHMIC CORRECTIONS which arise because we considered MASSLESS PARTICLES in the loop.

As explained in 2305.10490, around eq 2.14, for MASSIVE PARTICLES in the loop, we have a modification to the propagator $\sim \frac{m^2}{-p^2}$. More details can be found in gr-qc/9405057, where it is explained that it arises as a correction to the vertex



Eventually, one has

$$\pi^{-1}(p^2) = p^2 \left(1 - \frac{N p^2}{120 M_p^2} \log\left(-\frac{p^2}{\mu^2}\right) + \gamma \sum_{m=1}^{N} \frac{p^2}{M_p^2} \frac{m_n}{\sqrt{-p^2}} \right)$$

For $\mu \sim \Lambda_{sp}$ the log-term vanishes. Then, one can check that for $m_n = \{$KK or string spectrum$\}$ one arrives at

TREE LEVEL = 1-LOOP when $\Lambda^2_{sp} = \frac{M_p^2}{N}$

WITHOUT LOG CORRECTIONS! From this perspective, the log-corrections are just an accident of the purely massless case. In string theory we do have MASSIVE modes, so we would NOT expect these MULTIPLICATIVE LOG-CORRECTIONS. Instead, we expect ADDITIVE LOG-CORRECTIONS, as we will explain later.

Up to this point, we provided evidence for a scale

$$\Lambda_{sp} = \frac{M_p}{N^{\frac{1}{d-2}}} < M_p \qquad \text{SPECIES SCALE}$$

at which PERTURBATIVE GRAVITY breaks down.
Now, we would like to provide NON-PERTURBATIVE arguments to arrive at the same conclusion. This will complete the picture and support the idea that the SPECIES SCALE is the scale at which GRAVITY BECOMES STRONGLY COUPLED and as such it gives an upper bound on the UV cutoff of gravitational EFTs.

Arguably, the first NON-PERTURBATIVE argument was provided in 0706.2050, 0710.4344 and it runs as follows.
(See page 46 of 1903.06239)
Consider N bosonic species of mass Λ_{sp} and each of them with a gauged Z_2 symmetry. The system has thus Z_2^N gauged symmetry. From these N species, we can form a BH of mass

$$M_{BH} \simeq N \cdot \Lambda_{sp}$$

As usual, the temperature of the BH is

$$T_{BH} = \frac{1}{R_{BH}} \simeq \left(\frac{M_p^{d-2}}{M_{BH}}\right)^{\frac{1}{d-3}}$$

i.e. $$M_{BH} \sim \frac{M_p^{d-2}}{T_{BH}^{d-3}}$$

Since the $\mathbb{Z}_2^N$ symmetry is gauged, it must be revealed during evaporation. However, initially $\Lambda_{sp} \gg T_{BH}$ and the emission of the $\mathbb{Z}_2^N$-charged particles is suppressed by a Boltzmann factor

$$e^{-\Lambda_{sp}/T_{BH}}$$

By radiating other particles, the <u>BH WILL DECREASE ITS RADIUS</u> R_{BH} (hence increase its T_{BH}), until

$$\frac{1}{R_{BH}} \sim T_{BH} \sim \Lambda_{sp} \equiv \frac{1}{R_{BH,\min}}$$

and its mass is

$$M_{BH} \sim \frac{M_P^{d-2}}{\Lambda_{sp}^{d-3}}$$

At this point, the $\mathbb{Z}_2^N$-charged particles can be emitted. However, there are at most N of them, thus, we have

$$N \Lambda_{sp} \sim M_{BH} \sim \frac{M_P^{d-2}}{\Lambda_{sp}^{d-3}} \qquad N \sim \left(\frac{M_P}{\Lambda_{sp}}\right)^{d-2} \sim S_{BH}$$

from which $$\boxed{\Lambda_{sp} = \frac{M_P}{N^{\frac{1}{d-2}}}}$$

<u>LESSON</u>:

- $\Lambda_{sp} = \frac{M_P}{N^{\frac{1}{d-2}}} < M_P$ is the scale at which gravity becomes strongly coupled.
- It gives an UPPER BOUND on the UV cutoff of gravitational EFTs.
- It is defined as the (inverse) size of the smallest possible BH in the theory

$$R_{BH,\min} \sim \Lambda_{sp}^{-1}$$

- THE SPECIES SCALE IN SOME SIMPLE EXAMPLES

We have seen that in a theory of gravity with N particle species, the UV cutoff is

$$\Lambda_{sp} = \frac{M_p}{N^{\frac{1}{d-2}}}$$

We want to see how this relation is realized concretely in simple setups. The idea will be to COUNT ALL STATES whose mass is up to Λ_{sp}.

Clearly, in a general setup this is not an easy task. However, we can use SWAMPLAND CONJECTURES as an organizing principle and extract some general lesson. The swampland DISTANCE CONJECTURE, and its refinements in particular the EMERGENT STRING CONJECTURE (1910.01135), tell us that natural candidates for species are KK-MODES and STRING MODES. Therefore, we can study simple setup involving these modes and calculate Λ_{sp} therein.

• SPECIES as KK-MODES

Let us consider a theory with a tower of KK states and assume these are the only species (up to $O(1)$ other states in the theory). We want to understand at which scale gravity becomes strongly coupled.

The KK-states are organized as a tower

IT CHANGES FOR STRINGS

$$M_{KK,n} = n^{1} m_{KK} = n \frac{\hat{M}_p}{R}$$

M_p of $(d+1)$ dim. theory

The number of KK modes up to Λ_{sp} is thus

$$\frac{\Lambda_{sp}}{m_{KK}} = N = \frac{M_p^{d-2}}{\Lambda_{sp}^{d-2}}$$

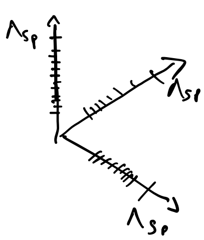
the KK mode sees the higher-dim theory

hence

$$\Lambda_{sp}^{d-1} = m_{KK} M_p^{d-2} = \qquad \hat{M}_p^{d-2} = \frac{M_p^{d-2}}{R}$$

$$= \frac{\hat{M}_p}{R} M_p^{d-2} \equiv \hat{M}_p^{d-1} \qquad \left[\hat{M}_p^{d-2} \int \hat{R} = \hat{M}_p^{d-2} Vol \int R \equiv M_p^{d-2} \int R \right]$$

This can be generalized to multiple (independent) towers



$$\left(\frac{\Lambda_{sp}}{m_{KK}}\right) \times \left(\frac{\Lambda_{sp}}{m_{KK}}\right) \times \dots \times \left(\frac{\Lambda_{sp}}{m_{KK}}\right) = \left(\frac{\Lambda_{sp}}{m_{KK}}\right)^{k}$$

$$\parallel$$

$$N \times N \times \dots \times N$$

$$\parallel$$

$$N_{TOT} \leftarrow N_{sp}$$

$$\parallel$$

$$\left(\frac{M_p}{\Lambda_{sp}}\right)^{d-2}$$

Using $\begin{cases} m_{KK} = \dfrac{\hat{M}_p}{\mathrm{Vol}^{\frac{1}{k}}} \\ M_p^{d-2} = \hat{M}_p^{d-2}\,\mathrm{Vol} \end{cases}$

we get $\Lambda_{sp}^{d+k-2} \simeq \hat{M}_p^{d+k-2}$ i.e. $\boxed{\Lambda_{sp}\big|_{KK} \simeq \hat{M}_p}$

A nice complementary argument involving COUNTING of BH microstates has been provided in 2305.10490, appendix B. The idea is to count the number of multiparticle states whose total mass is equal to that of the BH. Assuming we have at our disposal only a tower of KK-states with mass spacing m_{KK}, we have

$$N = \frac{\Lambda_{sp}}{m_{KK}} \quad \#\text{ of KK states up to } \Lambda_{sp}$$

$$K = \frac{M_{BH}}{m_{KK}} \quad \#\text{ of multiparticle states up to } M_{BH}$$

with $K \gg N$ (since $M_{BH} \simeq N \Lambda_{sp}$)

indeed $K = \dfrac{M_{BH}}{m_{KK}} = N \dfrac{\Lambda_{sp}}{m_{KK}} = N^2$

the number of BH microstates is given by the number of partitions of K into N integers, $\Omega_{K,N}$. Since $K \gg N$, we can approximate this number by

$$e^{S_{BH}} := \Omega_{K,N} \simeq \frac{K^{N-1}}{(N-1)!\,N!} \sim \frac{K^N}{(N!)^2} \overset{K\sim N^2}{\sim} \frac{N^{2N}}{(N!)^2} \sim e^{2N}$$

i.e. $S_{BH} = N$ ✓

LESSON:

- Gravity is NOT STRONGLY COUPLED at m_{KK}, but at $\Lambda_{sp} \simeq \hat{M}_p = \left(\mathrm{vol}^{\frac{1}{k}}\, m_{KK}\right) \gg m_{KK}$

- At m_{KK} I just have a higher-dimensional theory of weakly coupled gravity

- The species scale for a decompactification limit from $d \to d+k$ is given by (for independent towers)

$$\left(\frac{\Lambda_{sp}}{m_{KK}}\right)^{k} = N = \left(\frac{M_p}{\Lambda_{sp}}\right)^{d-2}$$

$$\Lambda_{sp}^{d+k-2} = m_{KK}^{k}\, M_p^{d-2}$$

$$\boxed{\Lambda_{sp} = m_{KK}^{\frac{k}{d+k-2}}\, M_p^{\frac{d-2}{d+k-2}}}$$

- For more complicated towers, see general algorithm given in 2112.10796

- SPECIES as STRING MODES

This case is a bit more subtle. First, it is not clear if it makes sense to use a QFT approach as that of the graviton propagator. If we try nevertheless to do so, following page 9 of 2212.03908, we have

STRING SPECTRUM $\quad M_m = \sqrt{m}\, M_s$ at $m = m_{MAX}$

$$\Lambda_{sp} = \frac{M_p}{N^{\frac{1}{d-2}}} \equiv M_s \sqrt{m_{MAX}}$$

but at level m there is a number of states (for $m \gg 1$)

$$d_m \simeq \gamma\, m^{-\frac{\nu}{2}}\, e^{\beta\sqrt{m}}$$

Thus, the number of species is

$$N = \sum_{m=1}^{m_{MAX}} d_m$$

We want to find $m_{MAX} = m_{MAX}(N, M_s)$ and then substitute it into the above relation $\Lambda_{sp} = M_s \sqrt{m_{MAX}}$ to read off $\Lambda_{sp} = \Lambda_{sp}(N, M_s)$.

To this purpose, we can write

$$N^{\frac{1}{d-2}} \frac{\Lambda_{sp}}{M_s} = \boxed{\frac{M_p}{M_s} = \underbrace{N}_{\sum_{m=1}^{m_{MAX}} d_m}{}^{\frac{1}{d-2}} \sqrt{m_{MAX}}}$$

This is an equation in m_{MAX} which can be solved to find (for $\frac{M_s}{M_p} \to 0$)

$$\sqrt{m_{MAX}} \sim (d-2) \log\Big(\overbrace{\frac{M_p}{M_s}}^{\sigma}\Big) + \mathcal{O}\left(\log\log\left(\frac{M_p}{M_s}\right)\right)$$

Putting this back into $\Lambda_{sp} = M_s \sqrt{m_{MAX}}$, we get

$$\Lambda_{sp} \simeq M_s \log\Big(\overbrace{\frac{M_p}{M_s}}^{\sigma}\Big) \overset{!}{>>} M_s$$

We get that Λ_{sp} is LARGER THAN M_s!

Now, let's try with the BH argument, following 2305.10490, section 3.1. For a BH of mass M_{BH} made up of string states, we have $(d=4)$

$$\boxed{\sqrt{m_{MAX}} = \frac{M_{BH}}{M_s} = \frac{M_p^2}{M_s \Lambda_{sp}}} = \frac{M_p}{M_s} N^{\frac{1}{d-2}} \quad \left[\begin{array}{l} M_{BH} = \dfrac{M_p^2}{\Lambda_{sp}} \\ S_{BH} = \left(\dfrac{M_p}{\Lambda_{sp}}\right)^2 \end{array}\right]$$

On the other hand, the BH entropy is

$$N :\overset{(*)}{=} S_{BH} = \log d_{m_{MAX}} = \sqrt{m_{MAX}} - \frac{\nu}{\beta} \log \sqrt{m_{MAX}}$$

$$\equiv \left(\frac{M_p}{\Lambda_{sp}}\right)^2$$

DIFFERENT STEP WRT QFT APPROACH

Combining these two equations in order to eliminate m_{MAX}, we get $\left(\text{for } \frac{M_s}{M_p} \to 0\right)$

$\sigma = \frac{M_p}{M_s}$

$$\Lambda_{sp} \simeq M_s + \frac{\nu}{\beta} M_s \frac{1}{\sigma} \log \sigma + \cdots$$

For $\sigma \to \infty$ $\left(\frac{M_s}{M_p} \to 0\right)$, we have

$$\boxed{\Lambda_{sp} \simeq M_s}$$

Thus, the BH approach gives a different result with respect to the QFT approach

• SPECIES SCALE AND HIGHER-DERIVATIVE CORRECTIONS

Given that the QFT and BH approach to the species scale seem to give different answers, it might be useful to have yet another method to compute the species scale. We can do so by looking at the derivative expansion (of the gravitational sector) of a spacetime effective action

any combination of 2 Riemann

$$\mathcal{L} \simeq M_p^{d-2}\left[R + \frac{C_2}{M_p^{d-2}} R^2 + \dots\right] \qquad \begin{array}{lll} [C_2] = d-4 & (\text{mass}) \\ [R] = 2 & (\text{mass}) \end{array}$$

$$\simeq M_p^{d-2}\Lambda_{sp}^2\left[R_{(0)} + \frac{C_{2,(0)}\Lambda_{sp}^{d-2}}{M_p^{d-2}} R_{(0)}^2 + \dots\right]$$

where we introduced dimensionless quantities

$$C_{2,(0)} = \frac{C_2}{\Lambda_{sp}^{d-4}} \qquad R_{(0)}^2 = \frac{R}{\Lambda_{sp}^2}$$

The EFT breaks down in the gravitational sector when the coefficient of $R^2_{(0)}$ is of order 1. This happens at a scale

$$\Lambda_{sp}^{d-2} = \frac{M_p^{d-2}}{C_{2,(0)}} \iff C_{2,(0)} \equiv N$$

$\Rightarrow$ We learn that the SPECIES SCALE is the scale in front of the R^2 term in the spacetime effective action. Once this is identified, we can calculate the "number of species" as the ratio $N = \left(\frac{M_p}{\Lambda_{sp}}\right)^{d-2}$.

While N might not be a well-defined concept, Λ_{sp} is well defined as the scale multiplying R^2. The role of $R^{n>2}$ terms remain to be understood.

• SPECIES THERMODYNAMICS

We have seen that in a d-dimensional EFT, the "number of species" N is an INTENSIVE QUANTITY

$$N = \left(\frac{M_p}{\Lambda_{sp}} \right)^{d-2} = \left(M_p L_{sp} \right)^{d-2}$$

AREA, NOT VOLUME!

In (quantum) gravity, a well-known intensive quantity is the ENTROPY of a BH, since it is (roughly) given by the area of the horizon

$$S_{BH} = \left(M_p R_{BH} \right)^{d-2}$$

We have seen that N is basically set by the entropy of the SMALLEST BH in the EFT. Can we take this analogy any further? Can we develop a more complete thermodynamic picture in the language of species? This has been done in 2305.10489 and it is reviewed below. For simplicity, we consider a SCHWARZSCHILD BH, whose defining parameters are

$$\begin{cases} M_{BH} = \left(R_{BH} M_p \right)^{d-3} M_p = S_{BH}^{\frac{d-3}{d-2}} M_p \\ T_{BH} = \dfrac{1}{R_{BH}} \\ S_{BH} = \left(R_{BH} M_p \right)^{d-2} \end{cases}$$

One can check that

$$\boxed{S_{BH}\, T_{BH}^{d-2} = M_p^{d-2}}$$

or $$T_{BH} = \frac{M_p}{S_{BH}^{\frac{1}{d-2}}}$$

Given the similarity with

$$\Lambda_{sp} = \frac{M_p}{N^{\frac{1}{d-2}}}$$

we can propose a dictionary

$$S_{BH} \overset{=}{\longleftrightarrow} N \qquad \text{ENTROPY of SPECIES}$$

$$T_{BH} \overset{=}{\longleftrightarrow} \Lambda_{sp} \qquad \text{TEMPERATURE of SPECIES}$$

To complete the picture, we need to identify E s.t.

$$\frac{1}{T} = \frac{\partial S}{\partial E}$$

Since we have not used M_{BH} yet, a natural guess would be

$$M_{BH} \overset{=}{\longleftrightarrow} E$$

We can check this guess in concrete examples.

Let us consider again a tower of KK-modes as species, with

$$E_m = m\,\Delta E$$

$$\Delta E = \frac{\Lambda_{sp}}{N} = M_p\, N^{-\frac{d-1}{d-2}}$$

The total energy of the tower is

$$E = \sum_{m=1}^{N} E_m = \sum_{m=1}^{N} m \left(M_p\, N^{-\frac{d-1}{d-2}} \right) =$$

$$= M_p\, N^{-\frac{d-1}{d-2}} \cdot \overbrace{\frac{1}{2}\left(N^2 + N\right)}^{\sim N^2 \text{ for } N \gg 1} \simeq$$

$$\simeq M_p\, N^{-\frac{d-1}{d-2}}\, N^2 = M_p\, N^{\frac{d-3}{d-2}}$$

$$\equiv M_p\, S_{BH}^{\frac{d-3}{d-2}} = M_{BH} \quad ✓$$

The calculation can be generalized to a tower s.t.

$$E_m = m^{\frac{1}{p}}\, m_{TOWER}$$

$$\left(\text{clearly} \quad \Lambda_{sp} = N^{\frac{1}{p}}\, m_{TOWER} \right)$$

$p = 1$ KK

$p = \infty$ STRING

as explained at pg 5 of 2112.10796, $p \to \infty$ is needed to cancel any polynomial dependence, since the string degeneracy is exponential

Then we have

$$E = \sum_{m=1}^{N} E_m = m_{TOWER} \sum_{m=1}^{N} m^{\frac{1}{p}} =$$

$$= m_{TOWER} \cdot \underbrace{H}_{\text{GENERALIZED HARMONIC NUMBER}}\left[N, -\tfrac{1}{p}\right]$$

$$\overset{N \gg 1}{\simeq} \left[N^{\frac{1}{p}+1} \left(\frac{p}{1+p} + O\left(\frac{1}{N}\right) \right) + \zeta\left(-\frac{1}{p}\right) \right] m_{TOWER}$$

$\zeta(0) = -\frac{1}{2}$

$$= \underbrace{N^{\frac{1}{p}}\, m_{TOWER}}_{\Lambda_{sp}} \cdot \frac{p}{p+1} \cdot N \simeq \Lambda_{sp} \cdot \frac{p}{p+1} \cdot N$$

$$\overset{p \to \infty}{\simeq} \Lambda_{sp} \cdot N = M_p\, N^{\frac{d-3}{d-2}} \quad ✓$$

Therefore, we completed the dictionary

$$\begin{cases} S_{BH} \overset{=}{\longleftrightarrow} N & \text{ENTROPY of SPECIES} \\ T_{BH} \overset{=}{\longleftrightarrow} \Lambda_{sp} = M_p N^{-\frac{1}{d-2}} & \text{TEMPERATURE of SPECIES} \\ M_{BH} \overset{=}{\longleftrightarrow} E_{sp} = M_p N^{\frac{d-3}{d-2}} \end{cases}$$

and one can check that $\frac{1}{T} = \frac{\partial S}{\partial E}$

- **LAWS of SPECIES THERMODYNAMICS**

We have now a complete analogy between species and the thermodynamics of Schwarzschild BH. We can thus formulate the LAWS of SPECIES THERMODYNAMICS on the MODULI SPACE ($E_{sp} \equiv E$, $T_{sp} \equiv T$, $S_{sp} \equiv N$)
(see later)

- 0th LAW : Points with the same $\Lambda_{sp}(\phi)$ have the same $T_{sp}(\phi)$

- 1st LAW : $\delta E_{sp} = T_{sp}\, \delta S_{sp} + \Phi\, \delta Q + \dots$

- 2nd LAW : $\exists$ direction in moduli space s.t.

  $\delta \Lambda_{sp}(\phi) \leq 0$ i.e. $\delta S_{sp}(\phi) \geq 0$

- 3rd LAW : $T_{sp}(\phi) = 0$ is at INFINITE DISTANCE in moduli space.

- COMMENTS :

1) The 0^{th} law does NOT say that $\Lambda_{sp} = T_{sp}$. This happens for Schwarzschild BH but can be different for other BHs.

2) The interpretation of $\phi, Q, \dots$ in the 1^{st} LAW is still work in progress

3) The SDC implies the 2^{nd} LAW.
Viceversa 2^{nd} LAW + BHEDC implies the SDC

BHEDC (1912.07453)

In the limit $S_{BH} \to \infty$, there is a tower of light states with mass $m \simeq S_{BH}^{-\gamma} \to 0$, $\gamma \gtrsim O(1)$.

4) The HEAT CAPACITY of SPECIES is NEGATIVE

$$C_{sp} = \frac{\partial E_{sp}}{\partial T} = - \frac{d-3}{T_{sp}^{d-2}} < 0$$

the same happens for BHs: they get cooler when you add energy, why they get hotter if energy is taken away.
Given that $E_{sp} = M_p S_{sp}^{\frac{d-3}{d-2}}$ and that $\delta S_{sp} > 0$ towards the boundary of the moduli space, the DESERT POINT is the HOTTEST POINT of the moduli space: if I reduce $N = S_{sp}$, the energy decreases but the temperature increases since $C_{sp} < 0$.

• SPECIES SCALE IN STRING THEORY

So far we discussed the notion of SPECIES SCALE in QUANTUM GRAVITY, without really relying on STRING THEORY. Hence, the previous discussion is general and should apply to any theory of QUANTUM GRAVITY.
Since STRING THEORY is a theory of quantum gravity, it is meaningful to study the SPECIES SCALE within String Theory. Indeed, within string theory we can make more precise and quantitative investigations.

The starting point is the observation that, there being no free parameters in string theory, the NUMBER of SPECIES N (and thus also Λ_{sp}) must be a FUNCTION of the MODULI.
The first problem to be addressed is then how to determine

$$\Lambda_{sp} = \Lambda_{sp}(\phi)$$

$$N = N(\phi)$$

In general, we expect the answer to be MODEL-DEPENDENT.
However, we can try to get some intuition in simple cases.
To start, we observe that a function $N = N(\phi)$ can be derived just from dimensional reduction.
Indeed, it is known that for a compactification $\underset{10}{\underset{\shortparallel}{d}} \to \underset{10}{\underset{\shortparallel}{d}} - m$ we have

$$M_s = M_p \, \mathrm{Vol}_m^{-\frac{1}{d-2}} \, g_s^{\frac{2}{d-2}} \qquad g_s = e^{\phi}$$

m-dim volume in string units

or

$$\hat{M}_p = M_p \, \mathrm{Vol}_m^{-\frac{1}{d-2}}$$

By assuming $\Lambda_{sp} \equiv M_s$ or $\Lambda_{sp} \equiv \hat{M}_p$, we find

$$\Lambda_{sp} \equiv M_s = M_p \, \mathrm{Vol}_m^{-\frac{1}{d-2}} \, g_s^{\frac{2}{d-2}} \overset{!}{\equiv} \frac{M_p}{N^{\frac{1}{d-2}}}$$

$$\Lambda_{sp} \equiv \hat{M}_p = M_p \, \mathrm{Vol}_m^{-\frac{1}{d-2}} \overset{!}{\equiv} \frac{M_p}{N^{\frac{1}{d-2}}}$$

hence we get the MODULI-DEPENDENT EXPRESSION (in any d!)

$$\boxed{N(\phi) = \mathrm{Vol}_m \, g_s^{-2}}$$

Recall $g_s^2 = (R_{11} M_{p,11D})^3$

• MODULI-DEPENDENT SPECIES SCALE and BH ENTROPY

The result $N = N(\phi)$ can now be checked by looking at the entropy of the SMALLEST POSSIBLE BHs in string theory. To get an intuition, we can just concentrate on simple and well-understood examples. To this purpose, let us briefly review how to calculate the entropy of BPS BHs using the ATTRACTOR MECHANISM (hep-th/9602136). Concretely, we will look at BHs in 4d $N=2$ SUGRA. Here, there is a function of the VECTOR MULTIPLETS MODULI called CENTRAL CHARGE

$$Z = q_\Lambda L^\Lambda - p^\Lambda M_\Lambda = Z(t_i, \bar{t}_i) \qquad \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix} = e^{K/2} \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}$$

$$K = -\log\left(i(\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda)\right)$$

The ATTRACTOR MECHANISM states that the entropy of an extremal BH in the theory is given by the function

$$S_{BH} := \pi Z \bar{Z}$$

evaluated at the point t_* s.t. $\partial_i |Z| \Big|_{t=t_*} = 0$.

We will show how this can be exploited to get the correct moduli dependence $N = N(\phi)$ by looking at the smallest possible BH and identifying $S_{BH,min} \equiv N(\phi)$.

- BH IN HETEROTIC on $K_3 \times T^2$

We just need to know the central charge, which is

$$[F = -X^0 X^1] \qquad Z = \frac{1}{2}\left(g_s\, q - \frac{1}{g_s}\, p\right) \qquad \left(\begin{array}{c} \text{Hp} \quad q>0,\ p<0 \\ \text{for BPS BH} \end{array}\right)$$

where $g_s = e^{\phi}$ is a scalar in the vector multiplets.
The idea is to fix this scalar by extremizing the (modulus) of the central charge :

$$\partial_\phi |Z| = 0 \qquad \Longrightarrow \qquad g_s = g_s(p,q)$$

By a direct computation, one finds

$$g_s^2 = -\frac{p}{q} \qquad (\text{recall } \quad p<0)$$

This is the value of the scalar field at the horizon.
The entropy is then

$$S_{BH} = \frac{A}{4} := \pi\, \overbrace{Z\bar{Z}}^{= \frac{1}{4}(-pq - 2pq - pq) = -pq}\Big|_{hor} = -\pi\, pq \qquad > 0$$

This is the well-known computation giving the entropy of this class of BHs.
To extract the moduli-dependent function $N = N(\phi)$, we reverse engineer the last step.
Given

$$S_{BH} = -\pi\, pq\,,$$

and knowing $g_s^2 = -\frac{p}{q}$, we can replace one of the charges in terms of a scalar field. Which charge has to be replaced is determined by requiring that the expression $S_{BH} = S_{BH}(g_s)$ grows for $g_s \to 0$.

a) $p = -g_s^2 q \Rightarrow S_{BH} = +\pi g_s^2 q^2 \underset{\text{for } g_s \to 0}{\longrightarrow} 0$ ✗

b) $q = -p/g_s^2 \Rightarrow S_{BH} = +\pi \dfrac{p^2}{g_s^2} \underset{\text{for } g_s \to 0}{\longrightarrow} \infty$ ✓

Thus, we have to replace q and we get

$$S_{BH} = \pi \frac{p^2}{g_s^2}$$

Finally, the minimal entropy is found by minimizing the leftover charge(s). In this case, the minimal value is $p=1$, giving

$$S_{BH,\,min} \simeq g_s^{-2} = N(\Lambda)$$

matching with 0912.3167, formula 20.

[In that paper, $N \simeq g_s^{-2}$ is derived by imposing $T_{BH} \lesssim \Lambda_{sp} \equiv M_s$ and by recalling $M_s \simeq M_p \, Vol^{-\frac{1}{d-2}} g_s^{\frac{2}{d-2}}$, see pg 10]

Notice that in this setup the volume is in a hypermultiplet and thus it does NOT couple to the BH, while g_s is in a vectormultiplet and it enters the BH solution. In IIA the situation is the other way around. Therefore, this simple reasoning cannot capture the dependence of N from Vol in the heterotic frame. To capture it, we look at BHs in IIA.

• BHs in IIA on CY3

The vector multiplet sector is fixed by

$$F_0 = -\frac{1}{3!} C_{ijk} \frac{X^i X^j X^k}{X^0}$$

The CY volume is

$$V_{CY} = \frac{1}{3!} C_{ijk} t^i t^j t^k \qquad \left[\begin{array}{l} t^i = \mathrm{Im}\, z^i \\ z^i = \dfrac{X^i}{X^0} \end{array}\right]$$

We consider the central charge

$$Z = q_\Lambda L^\Lambda - p^\Lambda M_\Lambda \qquad = X^0 e^{\frac{K}{2}} \left(-q + \tfrac{1}{2} C_{ijk} z^i z^j p^k\right)$$

and a BH solution supported by $-q_0 = q$, $p^i > 0$ charges.
Microscopically, this is a D0 - D4 BH or a MSW BH in 5d.
One can show that the attractor equation $\partial_i |Z| = 0$ is solved by

$$p^\Lambda = i\,(X^\Lambda - \bar{X}^\Lambda)$$

$$q_\Lambda = i\,(F_\Lambda - \bar{F}_\Lambda)$$

The entropy is then

$$S_{BH} := \pi Z \bar{Z} \Big|_{hor} = 2\pi \sqrt{\frac{q}{6} C_{ijk} p^i p^j p^k}$$

To reverse-engineer the moduli dependence, we need to recall the expression of V_{CY} at the horizon, namely

$$V_{CY} = \sqrt{\frac{q^3}{\frac{1}{6} C_{ijk} p^i p^j p^k}}$$

Then, we follow the same logic as in the previous example. The only difference is that now we want the entropy to grow with the volume.

a) $\frac{1}{6} C_{ijk} p^i p^j p^k = \frac{q^3}{\mathcal{V}_{cy}^2} \Rightarrow S_{BH} \simeq 2\pi \frac{q^2}{\mathcal{V}_{cy}} \longrightarrow 0$ for $\mathcal{V}_{cy} \to \infty$ ✗

b) $q = \left(\frac{1}{6} C_{ijk} p^i p^j p^k\right)^{\frac{1}{3}} \mathcal{V}_{cy}^{2/3}$

$\hookrightarrow S_{BH} \simeq 2\pi \mathcal{V}_{cy}^{\frac{1}{3}} \left(\frac{1}{6} C_{ijk} p^i p^j p^k\right) \longrightarrow \infty$ for $\mathcal{V}_{cy} \to \infty$ ✓

So, the correct option is b) and the minimal entropy is obtained for:

$$\frac{1}{6} C_{ijk} p^i p^j p^k = 1 \Rightarrow S_{BH,min} \simeq \mathcal{V}_{cy}^{1/3} = \text{2-CYCLE VOLUME}$$

We learn that the species scale in this limit is governed by a 2-CYCLE VOLUME.
Interestingly, the same result holds also in the presence of higher-derivative corrections.
In fact, by including R^2 corrections we can confirm explicitly that the species scale (given by the scale multiplying the R^2-term) is governed by a 2-CYCLE volume in this setup.

• BHs in IIA on CY_3 WITH R^2-CORRECTION

In general, dealing with higher-derivatives corrections in SUGRA and string theory is not an easy task, especially if one aims at doing it in a manifestly supersymmetric way. However, one particular R^2 correction in the present setup is known. It descends from an R^4-correction in 11D and it is not renormalized due to anomaly cancellation. In 4d, it reads (see e.g. hep-th/9711053)

$$S_{corr} = \frac{1}{96\pi} \int c_{2i} \, \mathrm{Im}\, z^i \, \mathrm{Tr}\, R \wedge * R$$

$$\text{with} \qquad c_{2i} = \int_{CY_3} c_2(TCY_3) \wedge \omega_i$$

This correction can be supersymmetrized by modifying the previous prepotential as (hep-th/9812082) [REVIEW hep-th/0007195]

GRAVIPHOTON BACKGROUND

$$F_0(X) \longrightarrow F(X, A) \equiv F_0(X) + F_1(X) A$$

$$F_0(X) = -\frac{1}{6} \frac{C_{ijk} X^i X^j X^k}{X^0}$$

$$F_1(X) = d_i \frac{X^i}{X^0} \qquad ; \quad d_i = -\frac{1}{24} \frac{1}{64} c_{2i}$$

Interestingly, the attractor equations are the same as before (but $Z = Z(X, A)$). However, in the presence of higher-derivatives corrections, the Bekenstein-Hawking formula does NOT capture the full entropy. One should instead use the WALD FORMULA

$$S_{Wald} = 2\pi \oint_{S^2} \varepsilon_{ab}\,\varepsilon_{cd} \frac{\delta \mathcal{L}}{\delta R_{abcd}}$$

A direct computation gives

$$S_{BH} = \pi \left[Z\bar{Z} + 4\,\mathrm{Im}\,(A\,F_A) \right]$$

Inserting the specific prepotential for the model under investigation, results in

$$S_{BH} = 2\pi \sqrt{\frac{9}{6}\left(C_{ijk}p^i p^j p^k + c_{2i}p^i\right)}$$

The minimal entropy can now be achieved with

$$\frac{1}{6} C_{ijk} p^i p^j p^k = 0$$

giving

$$S_{BH,min} \simeq 2\pi \sqrt{\frac{9}{6} c_{2i}p^i} \simeq c_{2i}t^i \simeq F_1$$

i.e. $\boxed{N(\phi) \simeq F_1(t)}$

The number of species is counted by the correction F_1 to the tree-level prepotential F_0. Being linear in t, it represents again a 2-cycle volume.

Remarkably, the SAME FUNCTION OF THE MODULI appears also in a completely different context: it is the genus-one free energy of the topological string. Therefore, we have

$$Z_{top} = e^{\mathcal{F}_{top}} \qquad \mathcal{F}_{top} = \mathcal{F}_0 + \mathcal{F}_1 A + \dots$$

$$S = -\phi^\Lambda \frac{\partial \mathcal{J}}{\partial \phi^\Lambda} + \mathcal{J}$$

$$\mathcal{J} = 4\pi \, \mathrm{Im}\, F$$

$$Z_{BH} = e^{\mathcal{F}_{BH}} \qquad \mathcal{F}_{BH} = \text{legendre transf. of } S_{BH}$$

$$= \mathcal{F}_0 + F_1 A + \dots$$

Imposing that these functions be the same amounts to ask that

$$Z_{BH} = |Z_{top}|^2$$

This is known as OSV CONJECTURE (hep-th/0405146). Indeed that $N(\phi)$ is given by F_1 has been proposed in 2212.06841. It is justified by the fact that the topological string and the superstring are related as explained by hep-th/9307158.

• MODULAR INVARIANT SPECIES SCALE

We motivated the fact that the species scale is a function of the moduli and we determined this function in some simple examples.
However, one should keep in mind that the expressions we found are valid only in asymptotic regions of the moduli space, where e.g. we can trust the supergravity approximation. Indeed, the expressions we found should be understood as the asymptotic limit of a yet UNKNOWN function valid GLOBALLY over the moduli space.
In general, to find this function is hard.
In 2212.06841 it has been proposed that on the (vector multiplet) moduli space of type II string theory on CY_3, this function is related to the GENUS-ONE FREE ENERGY of the TOPOLOGICAL STRING.

In simple setups, such as toroidal orbifolds, we can reconstruct the full global expression of the species scale starting from its asymptotic form.
As explained in 2306.08673, this can be done by exploiting modular invariance, which is a duality on these manifolds. We explain this in simple examples for KK and string towers.
The starting point is the asymptotic expression

$$N = \mathcal{V}_K \, g_s^{-2}$$

- <u>KK - TOWERS</u> : The asymptotic expression that we found is

2-cycle

$$N = \mathcal{V}_6^{\frac{1}{3}} \simeq t \quad = \mathrm{Im}\, T$$

This diverges for $t \to \infty$. We want to replace it with a function of t which has the same divergence also for $t \to 0$. One can check that the appropriate function is

$$N \simeq \log |\eta(t)|^{-4}$$

This has the correct asymptotic behaviour and it is invariant under $t \to t+1$. To make it invariant also under $t \to -\frac{1}{t}$ we need to replace it with

$$N \simeq -\log \left[-i(T-\bar{T})\, |\eta(T)|^4 \right]$$

This function is valid over the full moduli space.

For $\mathrm{Im}\, T \to \infty$ we can write $\quad (\mathcal{V}_6 \simeq \mathrm{Im}\, T^3)$

$$N \simeq \mathcal{V}_6^{\frac{1}{3}} - 3 \log \mathcal{V}_6^{\frac{1}{3}}$$

Hence

$$\Lambda_{sp} = \frac{M_P}{\sqrt{N}} \simeq M_s \left(1 + \frac{3}{2} \mathcal{V}_6^{-\frac{1}{3}} \log \mathcal{V}_6^{\frac{1}{3}} \right)$$

$$> \frac{M_P}{\mathcal{V}_6^{\frac{1}{6}}} = M_s$$

- <u>STRING TOWERS</u> The logic is exactly the same as before.

The asymptotic expression is

$$\mathcal{N} \simeq g_s^{-2} \simeq \mathrm{Im}\, S$$

The function with the correct asymptotics is

$$\mathcal{N} = -\log |\eta(S)|^{12}$$

and its modular invariant completion is

$$\mathcal{N} = -\log \left[(\mathrm{Im}\, S)^3 |\eta(S)|^{12} \right]$$

For $\mathrm{Im}\, S \to \infty$, we have

$$\mathcal{N} \simeq g_s^{-2} - 3 \log g_s^{-2}$$

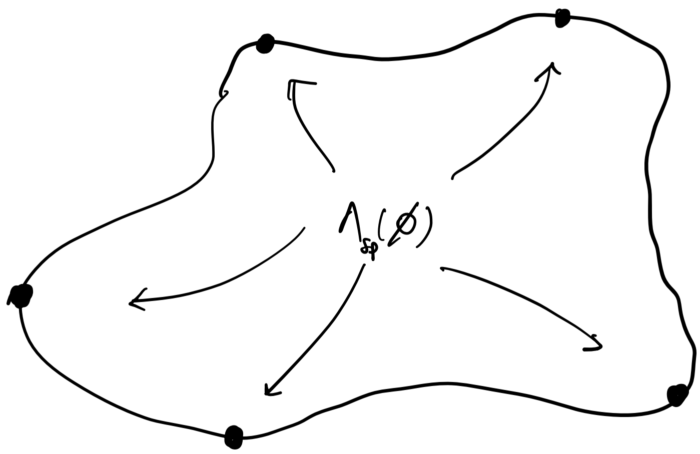
and thus

$$\Lambda_{sp} = \frac{M_P}{\sqrt{N}} \simeq M_s \left(1 + \frac{3}{2} g_s^2 \log g_s^{-2} \right)$$

$$\gtrsim g_s M_P = M_s$$

• OUTLOOK: SPECIES, EMERGENCE and DUALITIES

We gave strong evidence for the existence of a scale / function over the ENTIRE MODULI SPACE. This function contains precise information on the EFT, at all points in the moduli space. In particular, it tells us when the EFT breaks down.



At each point at the boundary we have a different asymptotic form of $\Lambda_{sp}(\phi)$. Hence, we have a different UV cutoff and a different spectrum content. In fact this suggests that there is a different EFT of QUANTUM GRAVITY.

In 2309.11551, 2309.11554 we proposed that

- this EFT can be quantized PERTURBATIVELY in $\frac{1}{N}$
- the fundamental d.o.f. are those infinite towers with mass spacing $< \Lambda_{sp}$.

Therefore, the SPECIES SCALE SERVES as a COMPASS across the MODULI SPACE of GRAVITATIONAL EFTs.