Numerical tests of the Eigenstate Thermalization Hypothesis

$$O_{mn} = O\left(\bar{E}\right)\delta_{mn} + e^{-S(\bar{E})/2} f_O\left(\bar{E},\omega\right) R_{mn}$$

$\bar{E} \equiv (E_m + E_n)/2, \omega \equiv E_n - E_m$ $\quad$ $S(E)$ -Thermodynamic entropy

$O\left(\bar{E}\right)$ smooth function =Microcanonical average

$f_O\left(\bar{E},\omega\right)$ smooth function, determines dynamics of observables

Eigenstate thermalization: a model

ETH holds already in relatively small quantum many-body systems

2d bosons, spin chains, 1d fermionic and bosonic systems

1d interacting bosons

$$\hat{H} = \sum_{j=1}^{L} \Big[-J \left(\hat{b}_j^\dagger \hat{b}_{j+1} + \text{H.c.} \right) + V \left(\hat{n}_j - \frac{1}{2} \right) \left(\hat{n}_{j+1} - \frac{1}{2} \right)$$
$$- J' \left(\hat{b}_j^\dagger \hat{b}_{j+2} + \text{H.c.} \right) + V' \left(\hat{n}_j - \frac{1}{2} \right) \left(\hat{n}_{j+2} - \frac{1}{2} \right) \Big]$$

$J' = V' = 0$ Integrable (by Bethe ansatz)

$J' \neq 0$ or $V' \neq 0$ Non-integrable, thermal

Diagonal matrix elements

<u>Observables:</u>

$$\hat{m}(k) = \frac{1}{L}\sum_{i,j} e^{ik(i-j)}\hat{b}_i^\dagger \hat{b}_j$$

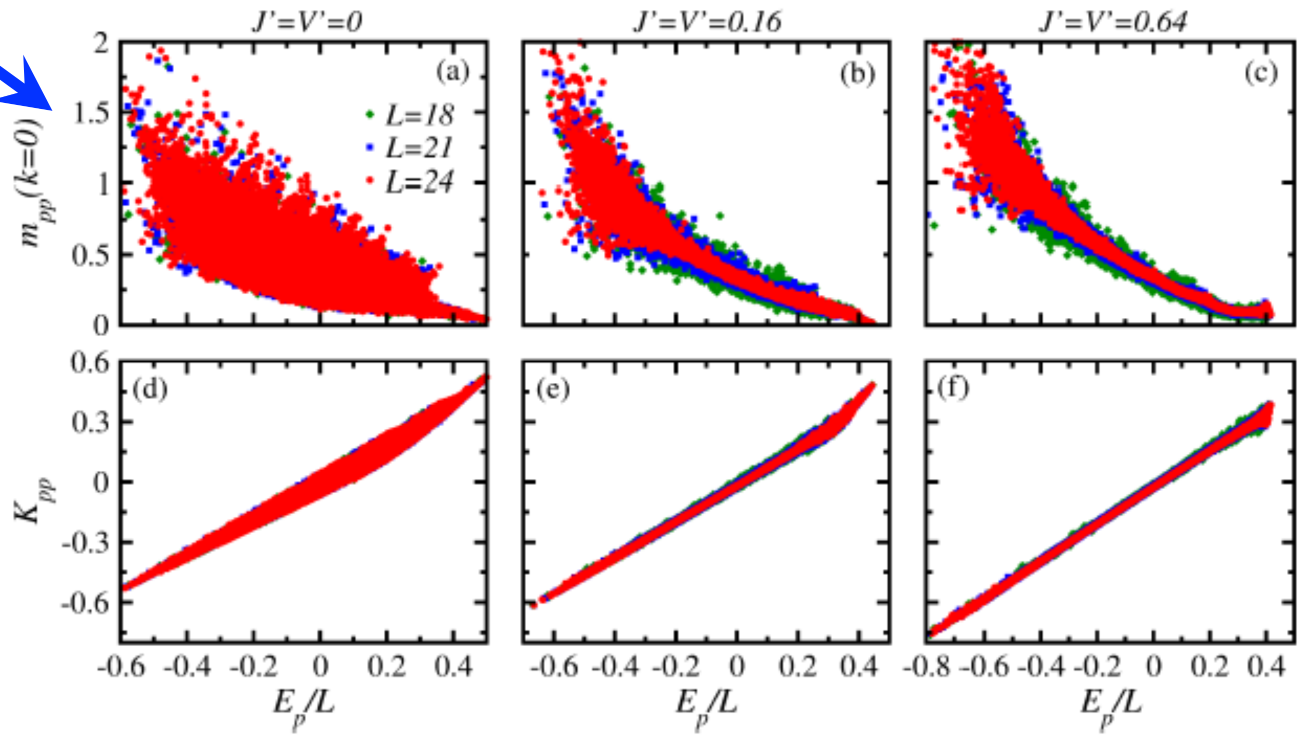
Zero-momentum mode occupation

$$\hat{K} = \frac{1}{L}\sum_{j=1}^{L}\left[-J\left(\hat{b}_j^\dagger \hat{b}_{j+1} + \text{H.c.}\right) - J'\left(\hat{b}_j^\dagger \hat{b}_{j+2} + \text{H.c.}\right)\right]$$

Kinetic energy per site

Integrable:

ETH violated



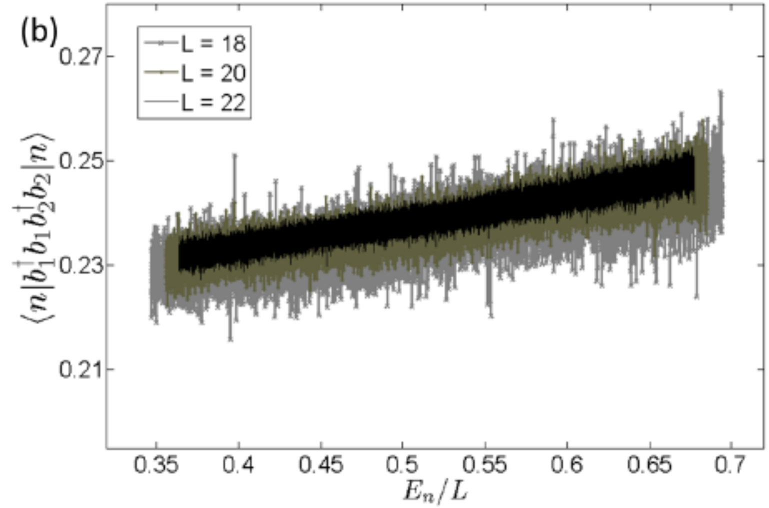
$N = L/3$

Diagonal observables vary smoothly; fluctuations decrease for larger systems

D'Alessio, Kafri, Polkovnikov, Rigol, AdvPhys'16

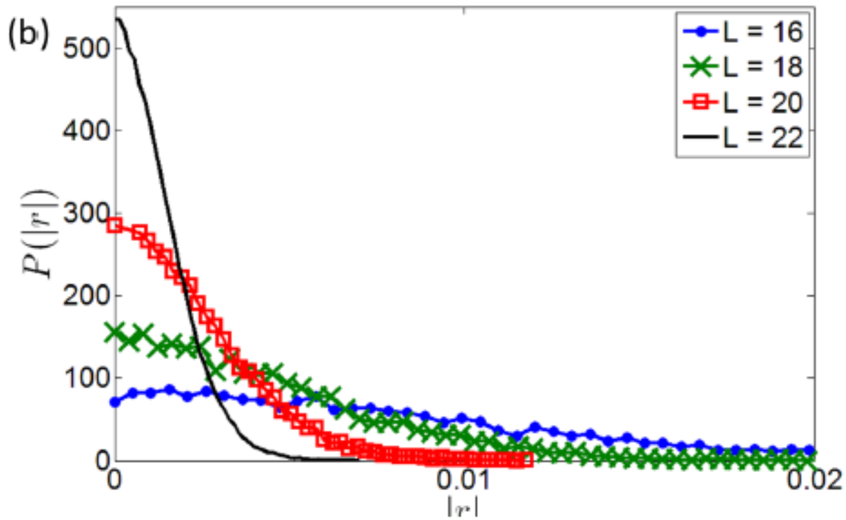
Diagonal matrix elements II

$N = L/2$ Observable: $\hat{O} = b_1^+ b_1 b_2^+ b_2$

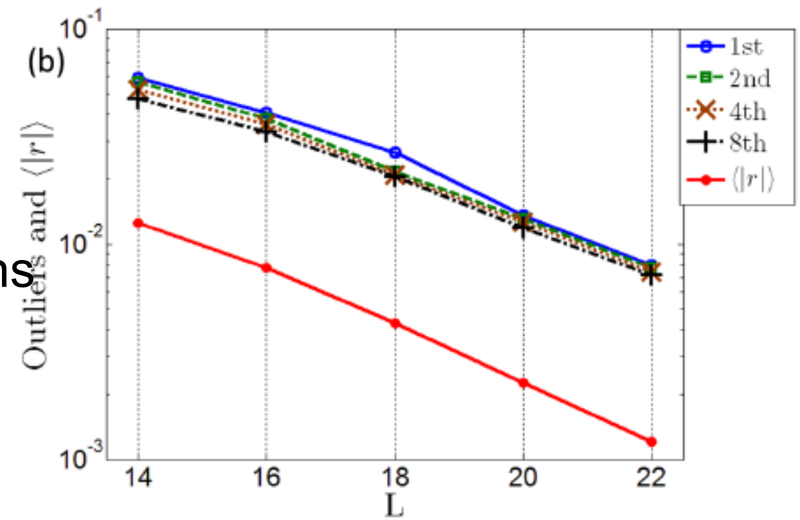


Distribution of

$$r_n = \langle n+1|\hat{O}|n+1\rangle - \langle n|\hat{O}|n\rangle$$



Outliers



For larger systems, fluctuations exponentially small

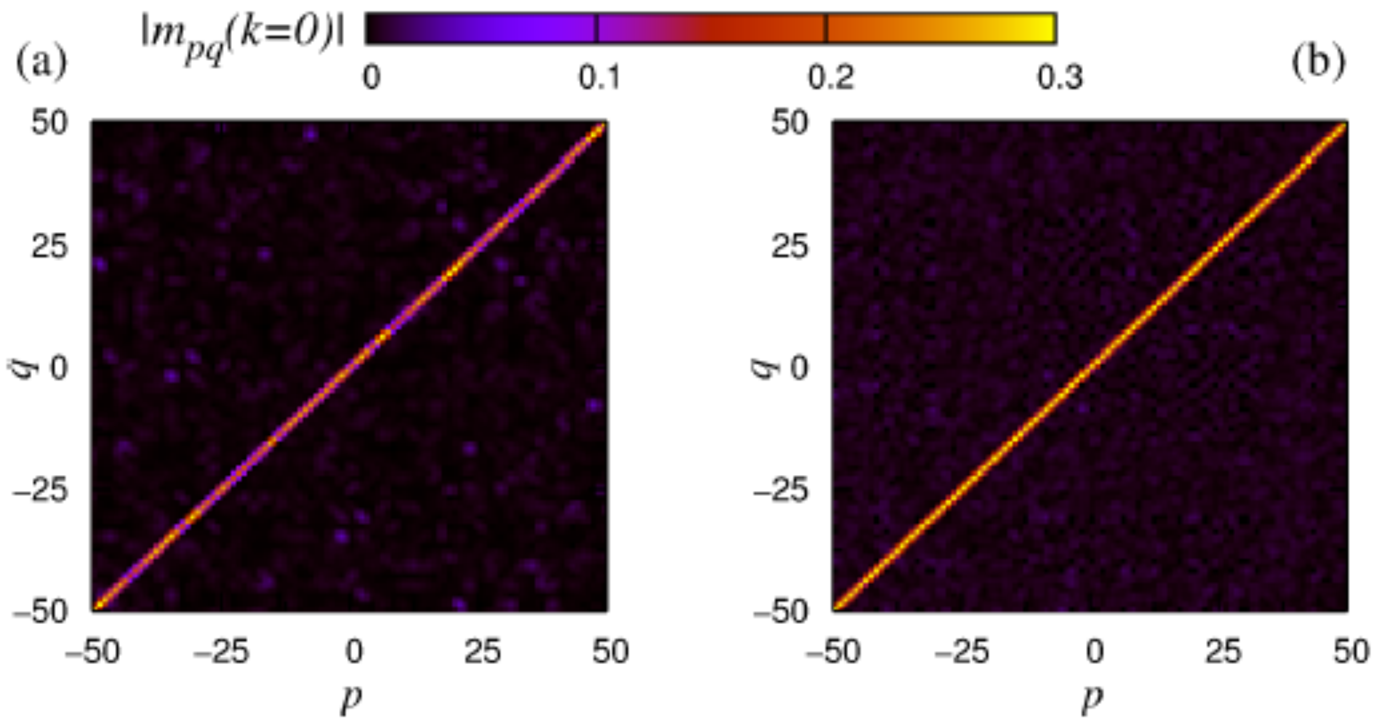
$$\langle r \rangle \sim e^{-CL}$$

Kim, Ikeda, Huse PRE'2015

Off-diagonal matrix elements

Zoom in onto a small energy window 100 eigenstates

Integrable   Non-integrable



Few off-diagonal matrix elements large

Many are zero

Nearly all matrix elements are small, of the same order

D'Alessio et al AdvPhys'16

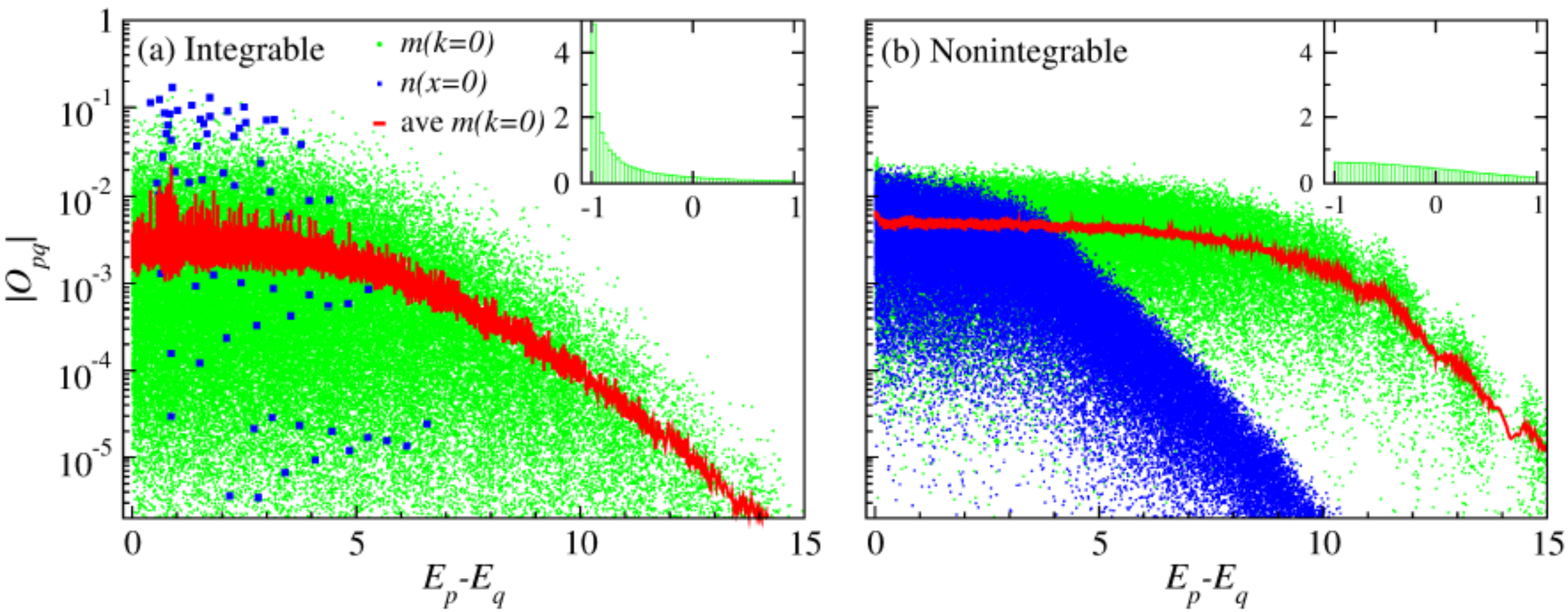
Off-diagonal matrix elements: energy dependence

$$\hat{H} = -J\sum_{j=1}^{L-1}\left(\hat{b}_j^\dagger \hat{b}_{j+1} + \text{H.c.}\right) + V\sum_{j<l}\frac{\hat{n}_j\hat{n}_l}{|j-l|^3} + g\sum_j x_j^2\,\hat{n}_j$$

$J=1 \qquad N=L/3$

dipolar int — trap potential

$g=1, V=0$ $\qquad$ $g=1, V=2$



Red lines are running averages (over 50 states)

$$O_{mn} = O\left(\bar{E}\right)\delta_{mn} + e^{-S(\bar{E})/2} f_O\left(\bar{E},\omega\right) R_{mn}$$

Carries information about transport, Thouless energy

Fast decay at large ω

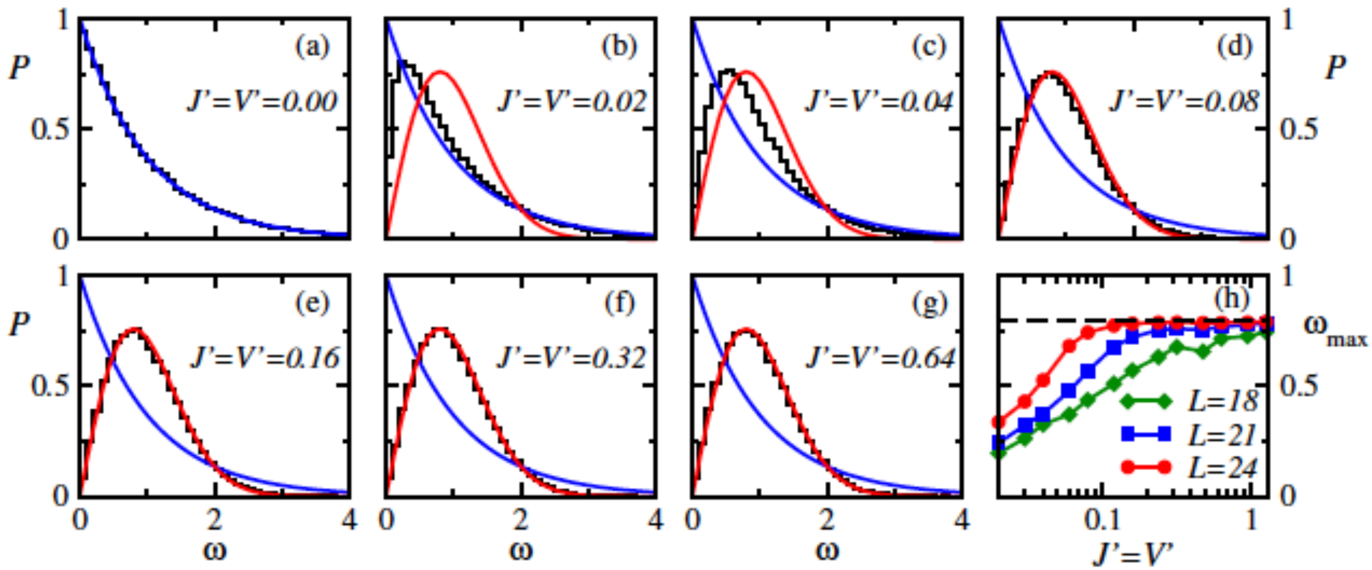
Khatami et al'13

Energy level statistics: Wigner-Dyson distribution

Model: 1d fermions

$$\hat{H} = \sum_{j=1}^{L} \left[-J \left(\hat{f}_j^\dagger \hat{f}_{j+1} + \text{H.c.} \right) + V \left(\hat{n}_j - \frac{1}{2} \right) \left(\hat{n}_{j+1} - \frac{1}{2} \right) - J' \left(\hat{f}_j^\dagger \hat{f}_{j+2} + \text{H.c.} \right) + V' \left(\hat{n}_j - \frac{1}{2} \right) \left(\hat{n}_{j+2} - \frac{1}{2} \right) \right]$$

$J' = V' = 0$ Integrable → Poisson level statistics $P(\omega) = \exp[-\omega]$



Broken integrability → Wigner-Dyson distribution

From Santos, Rigol'10